Intersecting chords

Worked solutions

GCSE Maths topic practice

This topic online

1 Chords AB and CD of a circle cross at the point X inside the circle.

AX = 3 cm, XB = 8 cm and CX = 4 cm.

Work out the length of XD.

(2 marks)

Internal chords: AX × XB = CX × XD

3 × 8 = 4 × XD

24 = 4 × XD

÷4 XD = 6 ÷4

XD = 6 cm

2 Chords AB and CD of a circle cross at the point X inside the circle.

AX = 5 cm, XB = 6 cm and XD = 4 cm.

Work out the length of CX.

(2 marks)

Internal chords: AX × XB = CX × XD

5 × 6 = CX × 4

30 = 4 × CX

÷4 CX = 7.5 ÷4

CX = 7.5 cm

3 Chords AB and CD of a circle, when extended, meet at the point P outside the circle. The points P, A, B lie in that order on one straight line and the points P, C, D lie in that order on the other.

PA = 4 cm, PB = 10 cm and PC = 5 cm.

Work out the length of PD.

(2 marks)

External chords: PA × PB = PC × PD

4 × 10 = 5 × PD

40 = 5 × PD

÷5 PD = 8 ÷5

PD = 8 cm

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4 Chords AB and CD of a circle, when extended, meet at the point P outside the circle. The points P, A, B lie in that order on one straight line and the points P, C, D lie in that order on the other.

PA = 3 cm, PB = 14 cm and PD = 12 cm.

Work out the length of PC. (2 marks)

External chords: PA x PB = PC x PD

3 x 14 = PC x 12

42 = 12 x PC

÷12 PC = 3.5 ÷12

PC = 3.5 cm

5 Chords AB and CD of a circle, when extended, meet at the point P outside the circle. The points P, A, B lie in that order on one straight line and the points P, C, D lie in that order on the other.

PA = 5 cm, AB = 7 cm and PC = 4 cm.

Work out the length of the chord CD. (3 marks)

PB is the whole line from P

PB = 5 + 7 = 12

External chords: PA x PB = PC x PD

5 x 12 = 4 x PD

60 = 4 x PD

÷4 PD = 15 ÷4

CD = 15 - 4

CD = 11 cm

6 A circular pond has four gates, A, B, C and D, on its edge.

A straight path from gate A to gate B and a straight path from gate C to gate D cross each other at the point X on the pond.

AX = 9 m, XB = 10 m and CX = 6 m.

Work out the total length of the path from C to D. Give your answer in metres. (3 marks)

The paths are chords: AX x XB = CX x XD

9 x 10 = 6 x XD

90 = 6 x XD

÷6 XD = 15 ÷6

CD = 6 + 15

21 m

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7 AB is a diameter of a circle. The chord CD crosses AB at the point X.

AX = 3 cm, CX = 4 cm and XD = 9 cm.

a. Work out the length of XB.

b. Work out the radius of the circle.

(3 marks)

a) A diameter is a chord: AX × XB = CX × XD

$3 \times XB = 4 \times 9$

$3 \times XB = 36$

$\div 3$ $\quad XB = 12$ $\quad \div 3$

$XB = 12$ cm

b)

$AB = 3 + 12 = 15$

$r = 15 \div 2$

7.5 cm

8 Chords AB and CD of a circle cross at the point X inside the circle.

$AX = x$ cm, $XB = (x + 3)$ cm, CX = 2 cm and XD = 9 cm.

Find the value of x.

(4 marks)

Internal chords: AX × XB = CX × XD

$x(x + 3) = 2 \times 9$

$x^2 + 3x = 18$

-18 $\quad x^2 + 3x - 18 = 0$ $\quad -18$

$(x + 6)(x - 3) = 0$

$x = -6$ or $x = 3$

A length cannot be negative

$x = 3$

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9 The straight lines AB and CD meet at the point P. The points P, A, B lie in that order on one of the lines and the points P, C, D lie in that order on the other.

PA = 5 cm, AB = 7 cm, PC = 6 cm and PD = 10 cm.

Lina says that the four points A, B, C and D all lie on the same circle.

Is Lina correct? You must show your working. (3 marks)

On a circle if $PA \times PB = PC \times PD$

$$PB = 5 + 7 = 12$$

$$PA \times PB = 5 \times 12 = 60$$

$$PC \times PD = 6 \times 10 = 60$$

Equal, so the chord property holds

Yes, Lina is correct: 60 = 60

10 Chords AB and CD of a circle, when extended, meet at the point P outside the circle. The points P, A, B lie in that order on one straight line and the points P, C, D lie in that order on the other.

PA = 3 cm, AB = 9 cm, $PC = x$ cm and $CD = (x + 1)$ cm.

Work out the length of PD. (5 marks)

Every length from P

$$PB = 3 + 9 = 12$$

$$PD = x + x + 1 = 2x + 1$$

External chords: $PA \times PB = PC \times PD$

$$3 \times 12 = x(2x + 1)$$

$$36 = 2x^2 + x$$

$$2x^2 + x - 36 = 0 \quad (-36)$$

$$(2x + 9)(x - 4) = 0$$

$x = -4.5$ or $x = 4$

A length cannot be negative, so $x = 4$

$$PD = 2 \times 4 + 1$$

PD = 9 cm

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