



- 1 ABCDEFGH is a cuboid with base ABCD and top face EFGH, where E is directly above A, F is directly above B, G is directly above C and H is directly above D.

AB = 3 cm, BC = 4 cm and CG = 12 cm.

Work out the length of AG.

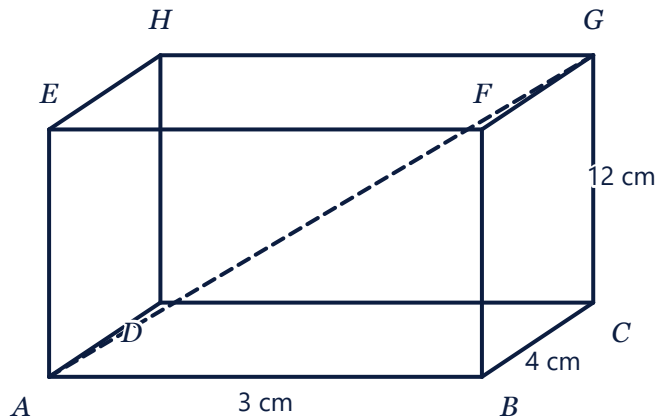


Diagram NOT accurately drawn

(2 marks)

First find the diagonal AC of the base using Pythagoras in triangle ABC: $AC^2 = 3^2 + 4^2 = 9 + 16 = 25$, so $AC = 5$ cm

AG is the space diagonal. Triangle ACG has a right angle at C because CG is vertical

$$AG^2 = AC^2 + CG^2 = 25 + 144 = 169$$

$$AG = \sqrt{169} = 13 \text{ cm}$$

Alternatively, in one step

$$AG = \sqrt{(3^2 + 4^2 + 12^2)} = \sqrt{169} = 13 \text{ cm}$$

13 cm

2

A cuboid measures 5 cm by 7 cm by 9 cm.

Calculate the length of the longest straight rod that will fit exactly inside the cuboid.

Give your answer correct to 1 decimal place.

(3 marks)

The longest rod that fits inside a cuboid lies along the space diagonal

$$\text{Space diagonal} = \sqrt{5^2 + 7^2 + 9^2} = \sqrt{25 + 49 + 81} = \sqrt{155}$$

$$\sqrt{155} = 12.4499..$$

The longest rod is 12.4 cm (1 decimal place)

12.4 cm

3

VABCD is a pyramid. The base ABCD is a horizontal square with sides of length 10 cm. The vertex V is directly above the centre O of the base. The height VO of the pyramid is 12 cm.

Calculate the length of the sloping edge VA. Give your answer correct to 3 significant figures.

(3 marks)

$$AC^2 = 10^2 + 10^2 = 200$$

$$AO^2 = \frac{200}{4} = 50$$

Triangle VOA, right angle at O

$$VA^2 = 12^2 + 50$$

$$VA^2 = 194$$

$$VA = 13.9 \text{ cm}$$

4

ABCDEFGH is a cuboid with base ABCD and top face EFGH, where E is directly above A, F is directly above B, G is directly above C and H is directly above D.

AB = 8 cm, BC = 6 cm and CG = 5 cm.

Calculate the size of the angle between the line AG and the plane ABCD.

Give your answer correct to 1 decimal place.

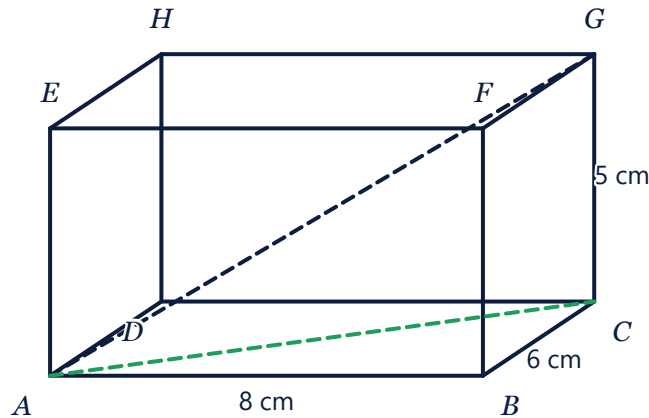


Diagram NOT accurately drawn

(4 marks)

A projection is the line's shadow on a plane, found by dropping perpendiculars to that plane. The angle between a line and a plane is measured between the line and this shadow. For two planes, use lines in each plane perpendicular to their common edge

The projection of AG onto the plane ABCD is AC, so the required angle is angle GAC

Find AC using Pythagoras in the base

$$AC^2 = 8^2 + 6^2 = 64 + 36 = 100, \text{ so } AC = 10 \text{ cm}$$

Triangle ACG has a right angle at C because CG is vertical

$$\tan(\text{angle GAC}) = CG \div AC = 5 \div 10 = 0.5$$

$$\text{angle GAC} = \tan^{-1}(0.5) = 26.565\dots^\circ$$

The angle between AG and the plane ABCD is 26.6° (1 decimal place)

26.6°

5

A storage box is a cuboid PQRSTUVW with base PQRS and top face TUVW, where T is directly above P, U is directly above Q, V is directly above R and W is directly above S.

PQ = 9 cm, QR = 4 cm and RV = 7 cm.

Work out the size of the angle that the line PV makes with the base PQRS.

Give your answer correct to 1 decimal place.

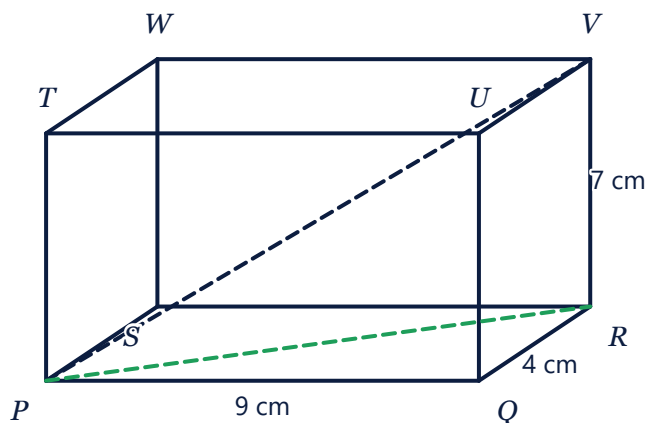


Diagram NOT accurately drawn

(4 marks)

A projection is the line's shadow on a plane, found by dropping perpendiculars to that plane. The angle between a line and a plane is measured between the line and this shadow. For two planes, use lines in each plane perpendicular to their common edge

The projection of PV onto the base is PR, so the required angle is angle VPR

Find PR using Pythagoras in the base

$$PR^2 = 9^2 + 4^2 = 81 + 16 = 97, \text{ so } PR = \sqrt{97} = 9.8488... \text{ cm}$$

Triangle PRV has a right angle at R because RV is vertical

$$\tan(\text{angle VPR}) = RV \div PR = 7 \div \sqrt{97} = 0.7107..$$

$$\text{angle VPR} = \tan^{-1}(0.7107...) = 35.40...^{\circ}$$

The angle is 35.4° (1 decimal place)

35.4°

6

A pyramid has a horizontal square base ABCD with sides of length 12 cm.

The vertex V is directly above the centre O of the base.

The height VO of the pyramid is 8 cm.

Calculate the size of the angle that the edge VA makes with the base ABCD.

Give your answer correct to 1 decimal place.

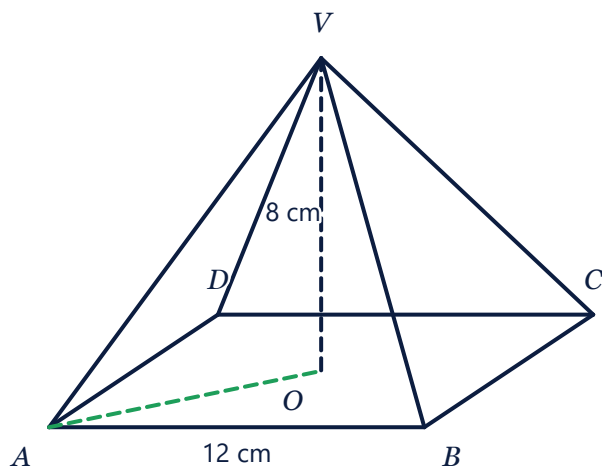


Diagram NOT accurately drawn

(4 marks)

A projection is the line's shadow on a plane, found by dropping perpendiculars to that plane. The angle between a line and a plane is measured between the line and this shadow. For two planes, use lines in each plane perpendicular to their common edge

The projection of VA onto the base is AO, so the required angle is angle VAO

AO is half the diagonal of the square base. Diagonal AC = $\sqrt{12^2 + 12^2} = \sqrt{288} = 12\sqrt{2}$ cm

So $AO = 6\sqrt{2} = 8.4852\dots$ cm

Triangle VAO has a right angle at O because VO is vertical

$\tan(\text{angle VAO}) = VO \div AO = 8 \div 6\sqrt{2} = 0.9428\dots$

$\text{angle VAO} = \tan^{-1}(0.9428\dots) = 43.31\dots^\circ$

The angle is 43.3° (1 decimal place)

43.3

7

The diagram shows a wedge ABCDEF.

The base ABCD is a horizontal rectangle with AB = 24 cm and BC = 10 cm.

E is vertically above D and F is vertically above C, with DE = CF = 6 cm.

ABFE is the sloping face of the wedge.

Work out the size of the angle between the line AF and the base ABCD.

Give your answer correct to 1 decimal place.

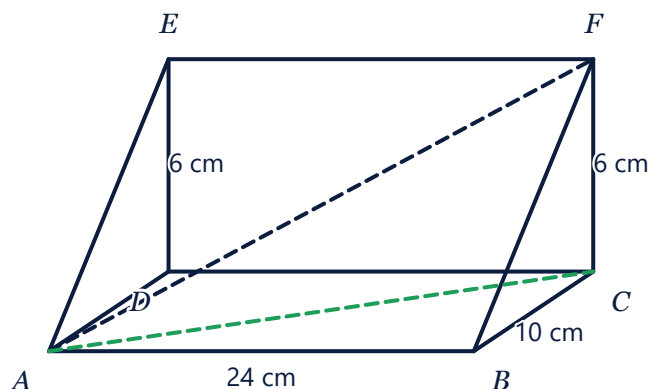


Diagram NOT accurately drawn

(4 marks)

A projection is the line's shadow on a plane, found by dropping perpendiculars to that plane. The angle between a line and a plane is measured between the line and this shadow. For two planes, use lines in each plane perpendicular to their common edge

F is vertically above C, so the projection of AF onto the base is AC and the required angle is angle FAC

Find AC using Pythagoras in the base rectangle

$$AC^2 = 24^2 + 10^2 = 576 + 100 = 676, \text{ so } AC = 26 \text{ cm}$$

Triangle ACF has a right angle at C because CF is vertical

$$\tan(\text{angle FAC}) = CF \div AC = 6 \div 26 = 0.2307..$$

$$\text{angle FAC} = \tan^{-1}(0.2307...) = 12.99...^{\circ}$$

The angle is 13.0° (1 decimal place)

13.0°

8

ABCDEFGH is a cuboid with base ABCD and top face EFGH, where E is directly above A, F is directly above B, G is directly above C and H is directly above D.

AB = 5 cm, BC = 12 cm and CG = 9 cm.

Calculate the size of the angle between the line AG and the plane BCGF.

Give your answer correct to 1 decimal place.

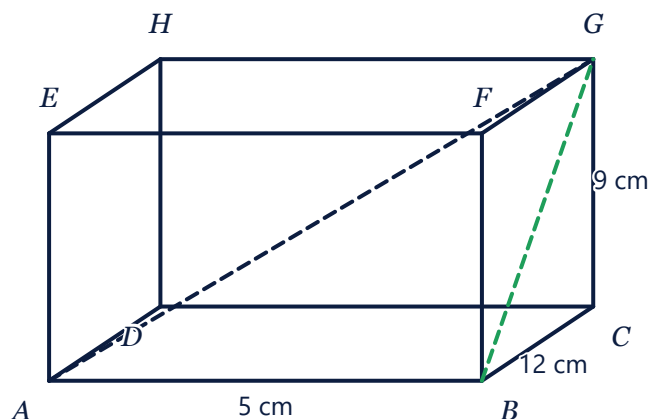


Diagram NOT accurately drawn

(4 marks)

A projection is the line's shadow on a plane, found by dropping perpendiculars to that plane. The angle between a line and a plane is measured between the line and this shadow. For two planes, use lines in each plane perpendicular to their common edge

AB is perpendicular to the plane BCGF, because AB is perpendicular to both BC and BF

So the projection of A onto the plane BCGF is B, the projection of AG onto the plane is BG, and the required angle is angle AGB

BG is a diagonal of the face BCGF

$$BG^2 = BC^2 + CG^2 = 12^2 + 9^2 = 144 + 81 = 225, \text{ so } BG = 15 \text{ cm}$$

Triangle ABG has a right angle at B because AB is perpendicular to the plane BCGF

$$\tan(\text{angle AGB}) = AB \div BG = 5 \div 15 = 0.3333..$$

$$\text{angle AGB} = \tan^{-1}\left(\frac{1}{3}\right) = 18.43...^\circ$$

The angle is 18.4° (1 decimal place)

18.4°

9

A pyramid has a horizontal square base ABCD with sides of length 16 cm.

The vertex V is directly above the centre O of the base.

The height VO of the pyramid is 15 cm.

M is the midpoint of BC.

Calculate the size of the angle between the face VBC and the base ABCD.

Give your answer correct to 1 decimal place.

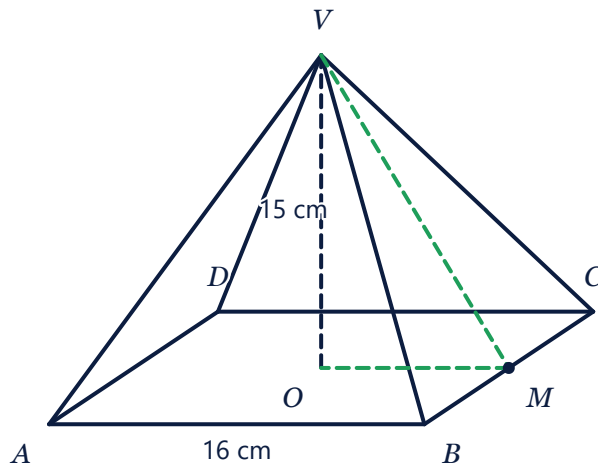


Diagram NOT accurately drawn

(4 marks)

A projection is the line's shadow on a plane, found by dropping perpendiculars to that plane. The angle between a line and a plane is measured between the line and this shadow. For two planes, use lines in each plane perpendicular to their common edge. The angle between the face VBC and the base is the angle VMO, where M is the midpoint of BC.

VM is perpendicular to BC (VBC is isosceles) and OM is perpendicular to BC, so angle VMO measures the angle between the two planes.

OM is the distance from the centre of the square to the midpoint of a side, so $OM = 16 \div 2 = 8 \text{ cm}$.

Triangle VOM has a right angle at O because VO is vertical.

$$\tan(\text{angle VMO}) = VO \div OM = 15 \div 8 = 1.875$$

$$\text{angle VMO} = \tan^{-1}(1.875) = 61.92\dots^\circ$$

The angle between the face VBC and the base is 61.9° (1 decimal place)

61.9°

10

A cuboid has a square base and a height of 8 cm.

The length of the longest diagonal of the cuboid, from one corner of the base to the opposite corner of the top face, is 12 cm.

Work out the volume of the cuboid.

(4 marks)

Let the side of the square base be x cm

The space diagonal of a cuboid satisfies $d^2 = x^2 + x^2 + h^2$

So $12^2 = x^2 + x^2 + 8^2$, which gives $144 = 2x^2 + 64$

$$2x^2 = 80, \text{ so } x^2 = 40$$

The volume is base area $\times$ height $= x^2 \times 8 = 40 \times 8 = 320$

Volume $= 320 \text{ cm}^3$. Note that the value of x itself is not needed, only x^2

320 cm^3

11

The diagram shows a tent in the shape of a prism.

The base ABCD is a horizontal rectangle with $AB = 16$ m and $BC = 6$ m.

The ridge EF is horizontal and runs parallel to AB, directly above the middle of the base, with E at the end above AD and F at the end above BC.

The ridge is 4 m above the base.

Work out the size of the angle between the line AF and the base ABCD.

Give your answer correct to 1 decimal place.

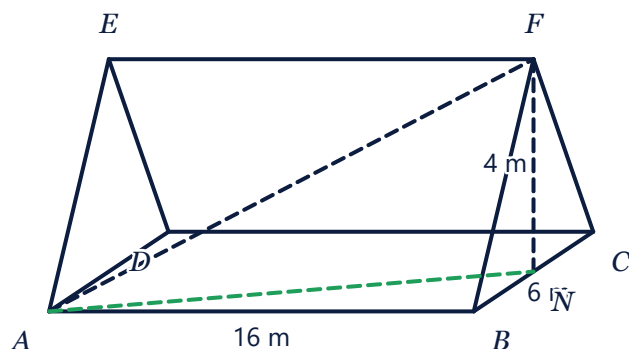


Diagram NOT accurately drawn

(5 marks)

A projection is the line's shadow on a plane, found by dropping perpendiculars to that plane. The angle between a line and a plane is measured between the line and this shadow. For two planes, use lines in each plane perpendicular to their common edge

Set up the projection of F onto the base. F is above the point N that lies on the base, on the line BC, midway between B and C, so N is 16 m from the line AD and 3 m from the line AB

The projection of AF onto the base is AN, so the required angle is angle FAN

Find AN using Pythagoras in the base

$$AN^2 = 16^2 + 3^2 = 256 + 9 = 265, \text{ so } AN = \sqrt{265} = 16.2788... \text{ m}$$

Triangle ANF has a right angle at N because FN is vertical, with $FN = 4$ m

$$\tan(\text{angle FAN}) = FN \div AN = 4 \div \sqrt{265} = 0.2457..$$

$$\text{angle FAN} = \tan^{-1}(0.2457...) = 13.80...^\circ$$

The angle between AF and the base is 13.8° (1 decimal place)

13.8