



1 Solve $x^2 - 5x - 14 > 0$

(3 marks)

$$= x^2 - 5x - 14 > 0$$

factorise

$$= (x - 7)(x + 2) > 0$$

critical values: $x = 7$ and $x = -2$

U-shaped graph is above the axis outside the roots

$$x < -2 \text{ or } x > 7$$

2 Solve $x^2 + 2x - 15 \leq 0$

Write your answer in set notation.

(3 marks)

$$= x^2 + 2x - 15 \leq 0$$

$$= (x + 5)(x - 3) \leq 0$$

critical values: $x = -5$ and $x = 3$

U-shaped graph is on or below the axis between the roots

$$\{x : -5 \leq x \leq 3\}$$

3 Solve $(x + 2)(x - 5) < 0$

(2 marks)

The graph of $y = (x + 2)(x - 5)$ is a U-shaped parabola with roots at $x = -2$ and $x = 5$

Between -2 and 5 , $x + 2$ is positive and $x - 5$ is negative, so their product is negative. Outside that interval the factors have the same sign, giving a positive product. At either endpoint the product is zero, which is excluded by < 0

The product is negative between the roots

$$\text{So } -2 < x < 5$$

$$-2 < x < 5$$

4 Solve $(x - 3)(x + 7) \geq 0$

(2 marks)

The roots of $(x - 3)(x + 7) = 0$ are $x = 3$ and $x = -7$

The parabola is U-shaped, so the product is greater than or equal to zero outside the roots (including the roots themselves because the inequality is not strict)

So $x = 3$

$$x = 3$$

5 The graph of $y = x^2 - 3x - 10$ is a U-shaped curve.

a. Factorise $x^2 - 3x - 10$ and write down where the curve crosses the x-axis.

b. Use a sketch of the curve to solve $x^2 - 3x - 10 \geq 0$.

(4 marks)

a)

$$x^2 - 3x - 10 = (x - 5)(x + 2)$$

curve crosses at $x = -2$ and $x = 5$

b) sketch a U-shape through -2 and 5

curve is on or above the axis outside the roots

$$x \leq -2 \text{ or } x \geq 5$$

6 Solve $x^2 - 9x + 18 < 0$

(3 marks)

Factorise

$$x^2 - 9x + 18 = (x - 3)(x - 6)$$

The roots are $x = 3$ and $x = 6$, and the parabola is U-shaped

The expression is negative between the roots, so $3 < x < 6$

$$3 < x < 6$$

7 Solve $x^2 + 3x - 28 \geq 0$

(3 marks)

Factorise

$$x^2 + 3x - 28 = (x + 7)(x - 4)$$

The roots are $x = -7$ and $x = 4$, and the parabola is U-shaped

The expression is greater than or equal to zero outside the roots

So $x = 4$

$$x = 4$$

8 Solve $x^2 + 12 < 7x$

(4 marks)

$$\begin{aligned} &= x^2 + 12 < 7x \\ -7x \quad &= x^2 - 7x + 12 < 0 \quad -7x \end{aligned}$$

factorise

$$= (x - 3)(x - 4) < 0$$

critical values 3 and 4; U-shape is below the axis between them

$$\boxed{3 < x < 4}$$

9 n is an integer such that $n^2 - n - 6 < 0$

Write down all the possible values of n .

(3 marks)

$$= n^2 - n - 6 < 0$$

$$= (n - 3)(n + 2) < 0$$

critical values -2 and 3; below the axis between them

$$= -2 < n < 3$$

n is an integer, so -2 and 3 are not included

$$\boxed{n = -1, 0, 1, 2}$$

10 n is an integer such that

$$n^2 - 2n - 15 \leq 0 \text{ and } 3n + 1 > 4$$

Find all the possible values of n .

(5 marks)

Factorise the quadratic

$$n^2 - 2n - 15 = (n - 5)(n + 3), \text{ so } (n - 5)(n + 3) \leq 0$$

$$\text{This gives } -3 \leq n \leq 5$$

Solve the linear inequality: $3n + 1 > 4$ gives $3n > 3$, so $n > 1$

The integers satisfying both -3 and 5 are 2, 3, 4 and 5

$$\boxed{2, 3, 4, 5}$$

11 Solve

$$5x - 4 > 2x + 8 \text{ and } x^2 - 10x + 21 < 0$$

Give your answer as a single inequality.

(5 marks)

Solve the linear inequality: $5x - 4 > 2x + 8$ gives $3x > 12$, so $x > 4$

Factorise the quadratic

$$x^2 - 10x + 21 = (x - 3)(x - 7), \text{ so } (x - 3)(x - 7) < 0$$

The expression is negative between the roots, so $3 < x < 7$

Both must hold: $x > 4$ combined with $3 < x < 7$ gives $4 < x < 7$

$$\boxed{4 < x < 7}$$

12 Solve $3 < x^2 - 6 \leq 19$

(5 marks)

Add 6 to every part

$$9 < x^2 \leq 25$$

From $x^2 > 9$: $x < -3$ or $x > 3$

$$\text{From } x^2 \leq 25: -5 \leq x \leq 5$$

Combine the two conditions on each side of zero

Answer

$$-5 \leq x < -3 \text{ or } 3 < x \leq 5$$

$$\boxed{-5 \leq x < -3 \text{ or } 3 < x \leq 5}$$

13 Solve $2x^2 - x - 15 < 0$

(4 marks)

Factorise

look for two numbers that multiply to $2 \times (-15) = -30$ and add to -1: they are -6 and 5

$$\begin{aligned} 2x^2 - 6x + 5x - 15 &= 2x(x - 3) + 5(x - 3) = \\ &= (2x + 5)(x - 3) \end{aligned}$$

The roots are $x = -\frac{5}{2}$ and $x = 3$. The graph of $y = 2x^2 - x - 15$ is a U-shaped parabola, which is below zero between its roots

So the solution is $-\frac{5}{2} < x < 3$. Check $x = 0$: $2(0) - 0 - 15 = -15 < 0$, correct

$$\boxed{-\frac{5}{2} < x < 3}$$

14 Solve $6x^2 - 7x - 5 > 0$

(4 marks)

$$= 6x^2 - 7x - 5 > 0$$

factorise

$$= (3x - 5)(2x + 1) > 0$$

critical values: $x = \frac{5}{3}$ and $x = -\frac{1}{2}$

U-shape is above the axis outside the roots

$$\boxed{x < -\frac{1}{2} \text{ or } x > \frac{5}{3}}$$

15 Solve $2x^2 + 5x - 3 \leq 0$

(5 marks)

Factorise

$$2x^2 + 5x - 3 = (2x - 1)(x + 3)$$

Check

$$(2x - 1)(x + 3) = 2x^2 + 6x - x - 3 = 2x^2 + 5x - 3$$

The roots are $x = \frac{1}{2}$ and $x = -3$, and the parabola is U-shaped

The expression is less than or equal to zero between the roots

$$\text{So } -3 \leq x \leq \frac{1}{2}$$

$$\boxed{-3 \leq x \leq \frac{1}{2}}$$

16 Solve

$$x^2 - x - 20 \leq 0 \text{ and } x^2 - 4 > 0$$

(5 marks)

Factorise the first quadratic

$$x^2 - x - 20 = (x - 5)(x + 4), \text{ so } (x - 5)(x + 4) \leq 0 \text{ gives}$$
$$-4 \leq x \leq 5$$

Factorise the second

$$x^2 - 4 = (x - 2)(x + 2), \text{ so } (x - 2)(x + 2) > 0 \text{ gives } x < -2 \text{ or } x > 2$$

Both must hold, so intersect $-4 \leq x < -2$ or $2 < x \leq 5$

Answer

$$-4 \leq x < -2 \text{ or } 2 < x \leq 5$$

$$\boxed{-4 \leq x < -2 \text{ or } 2 < x \leq 5}$$

17

A rectangular allotment bed has length $(x + 7)$ metres and width $(x - 2)$ metres, where $x > 2$.

The area of the bed is less than 90 m^2 .

a. Show that $x^2 + 5x - 104 < 0$

b. Find the range of possible values of x .

(6 marks)

a) Area = $(x + 7)(x - 2) = x^2 + 5x - 14$. The area is less than 90, so $x^2 + 5x - 14 < 90$

Subtract 90 from both sides: $x^2 + 5x - 104 < 0$, as required

b) Factorise

$$x^2 + 5x - 104 = (x + 13)(x - 8), \text{ since } 13 \text{ times } 8 = 104 \text{ and } 13 - 8 = 5$$

$$(x + 13)(x - 8) < 0 \text{ gives } -13 < x < 8$$

The question states $x > 2$ (the width must be positive), so the range is $2 < x < 8$