



- 1 x is a positive number. Show that $\frac{10}{\sqrt{5x}}$ can be written as $\frac{2\sqrt{5x}}{x}$.

(3 marks)

$$\frac{\{10\}}{\{\sqrt{5x}\}} = \frac{\{10 \times \sqrt{5x}\}}{\{\sqrt{5x} \times \sqrt{5x}\}} \\ = \frac{\{10\sqrt{5x}\}}{\{5x\}}$$

divide top and bottom by 5

$$= \frac{\{2\sqrt{5x}\}}{\{x\}}$$

$$\boxed{\frac{\{2\sqrt{5x}\}}{\{x\}}}$$

- 2 Rationalise the denominator of $\frac{12}{\sqrt{3}}$

Give your answer in its simplest form.

(2 marks)

Rationalise means remove the square root from the denominator. Multiplying top and bottom by $\sqrt{3}$ multiplies the fraction by $\frac{\sqrt{3}}{\sqrt{3}} = 1$, so it keeps the same value; $\sqrt{3} \times \sqrt{3} = 3$

Multiply the numerator and denominator by $\sqrt{3}$

$$\frac{12}{\sqrt{3}} = \frac{12\sqrt{3}}{3}$$

Divide top and bottom by 3

$$\frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

$$\boxed{4\sqrt{3}}$$

- 3 Erin is asked to rationalise the denominator of $\frac{8}{\sqrt{6}}$

Here is her working.

$$\frac{8}{\sqrt{6}} = \frac{8 \times \sqrt{6}}{\sqrt{6} \times \sqrt{6}} = \frac{8\sqrt{6}}{36}$$

Erin has made one mistake.

Explain the mistake Erin has made.

(2 marks)

Erin has squared 6 instead of squaring $\sqrt{6}$

$$\sqrt{6} \times \sqrt{6} = 6, \text{ not } 36$$

The correct answer is $\frac{8\sqrt{6}}{6}$, which simplifies to $\frac{4\sqrt{6}}{3}$

4 Write $\frac{6 + \sqrt{12}}{\sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are integers.

(4 marks)

$\sqrt{12} = 2\sqrt{3}$, so the numerator is $6 + 2\sqrt{3}$

Split the fraction: $\frac{6}{\sqrt{3}} + \frac{2\sqrt{3}}{\sqrt{3}}$

$$\frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}, \text{ and } \frac{2\sqrt{3}}{\sqrt{3}} = 2$$

So the expression equals $2 + 2\sqrt{3}$, giving $a = 2$ and $b = 2$

$$\boxed{2 + 2\sqrt{3}}$$

5 x is a number greater than 4. Rationalise the denominator of $\frac{6}{\sqrt{x} - 2}$. Give your answer as a single fraction.

(3 marks)

multiply by $(\sqrt{x} + 2)$ top and bottom

$$\{6\} / \{\sqrt{x} - 2\} = \{6(\sqrt{x} + 2)\} / \{(\sqrt{x} - 2)(\sqrt{x} + 2)\}$$

$$(\sqrt{x} - 2)(\sqrt{x} + 2) = x - 4$$

$$= \{6(\sqrt{x} + 2)\} / \{x - 4\}$$

$$\boxed{\{6(\sqrt{x} + 2)\} / \{x - 4\}}$$

6 Rationalise the denominator of $\frac{22}{4 + \sqrt{5}}$

Give your answer in the form $a + b\sqrt{5}$, where a and b are integers.

(4 marks)

Use the same two terms with the sign between them reversed (the conjugate). When the brackets multiply, the two square-root middle terms cancel, leaving a rational denominator. Multiply the numerator by the same bracket to keep the value unchanged

Multiply the numerator and denominator by $(4 - \sqrt{5})$

Denominator

$$(4 + \sqrt{5})(4 - \sqrt{5}) = 16 - 5 = 11$$

Numerator

$$22(4 - \sqrt{5}) = 88 - 22\sqrt{5}$$

So the fraction is $\frac{88 - 22\sqrt{5}}{11} = 8 - 2\sqrt{5}$, giving $a = 8$ and $b =$

-2

$$\boxed{8 - 2\sqrt{5}}$$

7

The first two terms of a geometric sequence are

$$\sqrt{5} + 1 \quad 5 + \sqrt{5}$$

Find the common ratio of the sequence.

Give your answer in its simplest form.

(4 marks)

The common ratio is the second term divided by the first:

$$\frac{5 + \sqrt{5}}{\sqrt{5} + 1}$$

Factorise the numerator

$$5 + \sqrt{5} = \sqrt{5}(\sqrt{5} + 1)$$

$$\text{So the ratio is } \frac{\sqrt{5}(\sqrt{5} + 1)}{\sqrt{5} + 1} = \sqrt{5}$$

$$\boxed{\sqrt{5}}$$

8

Show that $\sqrt{50} + \frac{18}{\sqrt{2}} = 14\sqrt{2}$

(5 marks)

$$\sqrt{50} = \sqrt{(25 \times 2)} = 5\sqrt{2}$$

Rationalise

$$\frac{18}{\sqrt{2}} = \frac{18\sqrt{2}}{2} = 9\sqrt{2}$$

$$5\sqrt{2} + 9\sqrt{2} = 14\sqrt{2}, \text{ as required}$$

9

Show that $\frac{7 + \sqrt{5}}{3 - \sqrt{5}}$ can be written as $\frac{13 + 5\sqrt{5}}{2}$

(5 marks)

Use the same two terms with the sign between them reversed (the conjugate).

When the brackets multiply, the two square-root middle terms cancel, leaving a rational denominator. Multiply the numerator by the same bracket to keep the value unchanged

Multiply the numerator and denominator by $(3 + \sqrt{5})$

Numerator

$$(7 + \sqrt{5})(3 + \sqrt{5}) = 21 + 7\sqrt{5} + 3\sqrt{5} + 5 = 26 + 10\sqrt{5}$$

Denominator

$$(3 - \sqrt{5})(3 + \sqrt{5}) = 9 - 5 = 4$$

So the fraction is $\frac{26 + 10\sqrt{5}}{4}$. Dividing top and bottom by 2 gives $\frac{13 + 5\sqrt{5}}{2}$, as required

- 10 Express $\frac{\sqrt{12} + 4}{\sqrt{3} - 1}$ in the form $a + b\sqrt{3}$, where a and b are integers.

Show each stage of your working.

(6 marks)

Use the same two terms with the sign between them reversed (the conjugate). When the brackets multiply, the two square-root middle terms cancel, leaving a rational denominator. Multiply the numerator by the same bracket to keep the value unchanged

Simplify the numerator

$$\sqrt{12} = 2\sqrt{3}, \text{ so the numerator is } 2\sqrt{3} + 4$$

Multiply the numerator and denominator by $(\sqrt{3} + 1)$

Numerator

$$(2\sqrt{3} + 4)(\sqrt{3} + 1) = 6 + 2\sqrt{3} + 4\sqrt{3} + 4 = 10 + 6\sqrt{3}$$

Denominator

$$(\sqrt{3} - 1)(\sqrt{3} + 1) = 3 - 1 = 2$$

So the expression is $\frac{10 + 6\sqrt{3}}{2} = 5 + 3\sqrt{3}$, giving $a = 5$ and $b =$

3

$$\boxed{5 + 3\sqrt{3}}$$

- 11 x is a number greater than 9. Rationalise the denominator of $\frac{\sqrt{x} + 3}{\sqrt{x} - 3}$. Give your answer in the form

$$x + \frac{a\sqrt{x} + b}{x - c}$$

(4 marks)

multiply by $(\sqrt{x} + 3)$ top and bottom

$$\frac{\{\sqrt{x} + 3\}}{\{\sqrt{x} - 3\}} = \frac{\{(\sqrt{x} + 3)(\sqrt{x} + 3)\}}{\{(\sqrt{x} - 3)(\sqrt{x} + 3)\}}$$

$$\text{top: } x + 3\sqrt{x} + 3\sqrt{x} + 9 = x + 6\sqrt{x} + 9$$

$$\text{bottom: } x - 9$$

$$\boxed{\frac{x + 6\sqrt{x} + 9}{x - 9}}$$