



- 1 Solve the simultaneous equations

$$y = x^2$$

$$y = x + 6$$

(4 marks)

Substitute $y = x^2$ into the second equation: $x^2 = x + 6$

Rearrange

$$x^2 - x - 6 = 0$$

Factorise

$$(x - 3)(x + 2) = 0, \text{ so } x = 3 \text{ or } x = -2$$

$$\text{When } x = 3, y = 9. \text{ When } x = -2, y = 4$$

$$\text{Check in } y = x + 6: 3 + 6 = 9 \text{ and } -2 + 6 = 4$$

$$x = 3, y = 9 \text{ and } x = -2, y = 4$$

- 2 Solve the simultaneous equations

$$y = x^2 - 3x$$

$$y = 2x - 6$$

(4 marks)

Set the expressions for y equal

$$x^2 - 3x = 2x - 6$$

Rearrange

$$x^2 - 5x + 6 = 0$$

Factorise

$$(x - 2)(x - 3) = 0, \text{ so } x = 2 \text{ or } x = 3$$

$$\text{When } x = 2, y = 2(2) - 6 = -2. \text{ When } x = 3, y = 2(3) - 6 = 0$$

Check in the quadratic

$$4 - 6 = -2 \text{ and } 9 - 9 = 0$$

$$x = 2, y = -2 \text{ and } x = 3, y = 0$$

3

Solve the simultaneous equations

$$y = x^2 - 4$$

$$3x - y = 6$$

(4 marks)

Make y the subject of the linear equation

$$y = 3x - 6$$

Both equal y , so set them equal

$$\begin{array}{rcl} x^2 - 4 & = & 3x - 6 \\ -3x + 6 & & -3x + 6 \\ x^2 - 3x + 2 & = & 0 \\ (x - 1)(x - 2) & = & 0 \end{array}$$

$$x = 1 \text{ or } x = 2$$

Substitute into $y = 3x - 6$

$$x = 1: y = 3 - 6 = -3$$

$$x = 2: y = 6 - 6 = 0$$

$$x = 1, y = -3 \text{ and } x = 2, y = 0$$

4

Solve the simultaneous equations

$$x^2 + y^2 = 20$$

$$y = 2x$$

(4 marks)

Substitute $y = 2x$ into the circle equation: $x^2 + 4x^2 = 20$

$$\text{So } 5x^2 = 20, \text{ giving } x^2 = 4$$

$$x = 2 \text{ or } x = -2$$

$$\text{When } x = 2, y = 4. \text{ When } x = -2, y = -4$$

Check

$$4 + 16 = 20 \text{ for both pairs}$$

$$x = 2, y = 4 \text{ and } x = -2, y = -4$$

5 Solve the simultaneous equations

$$y = x^2 + 2x - 1$$

$$y = 3x + 5$$

(4 marks)

Set the expressions for y equal

$$x^2 + 2x - 1 = 3x + 5$$

Rearrange

$$x^2 - x - 6 = 0$$

Factorise

$$(x - 3)(x + 2) = 0, \text{ so } x = 3 \text{ or } x = -2$$

$$\text{When } x = 3, y = 3(3) + 5 = 14. \text{ When } x = -2, y = 3(-2) + 5 = -1$$

Check in the quadratic

$$9 + 6 - 1 = 14 \text{ and } 4 - 4 - 1 = -1$$

$$x = 3, y = 14 \text{ and } x = -2, y = -1$$

6 Solve the simultaneous equations

$$x^2 + y^2 = 13$$

$$x + y = 5$$

(5 marks)

From the linear equation, $y = 5 - x$

Substitute

$$x^2 + (5 - x)^2 = 13$$

Expand

$$x^2 + 25 - 10x + x^2 = 13, \text{ so } 2x^2 - 10x + 12 = 0$$

Divide by 2

$$x^2 - 5x + 6 = 0, \text{ so } (x - 2)(x - 3) = 0 \text{ and } x = 2 \text{ or } x = 3$$

$$\text{When } x = 2, y = 3. \text{ When } x = 3, y = 2$$

Check

$$4 + 9 = 13 \text{ and } 9 + 4 = 13$$

$$x = 2, y = 3 \text{ and } x = 3, y = 2$$

7

Solve the simultaneous equations

$$y = 2x^2 - 3$$

$$y = 5x$$

(5 marks)

Set the expressions for y equal

$$2x^2 - 3 = 5x$$

Rearrange

$$2x^2 - 5x - 3 = 0$$

Factorise

$$(2x + 1)(x - 3) = 0, \text{ so } x = -\frac{1}{2} \text{ or } x = 3$$

$$\text{When } x = 3, y = 5(3) = 15. \text{ When } x = -\frac{1}{2}, y = 5(-\frac{1}{2}) = -\frac{5}{2}$$

Check in the quadratic

$$2(9) - 3 = 15 \text{ and } 2(\frac{1}{4}) - 3 = -\frac{5}{2}$$

$$x = 3, y = 15 \text{ and } x = -\frac{1}{2}, y = -\frac{5}{2}$$

8

The circle **C** has equation $x^2 + y^2 = 10$ The line **L** has equation $y = x - 2$ Find the coordinates of the points where **L** intersects **C**.

(5 marks)

$$\text{Substitute } y = x - 2 \text{ into the circle equation: } x^2 + (x - 2)^2 = 10$$

Expand

$$x^2 + x^2 - 4x + 4 = 10, \text{ so } 2x^2 - 4x - 6 = 0$$

Divide by 2

$$x^2 - 2x - 3 = 0, \text{ so } (x - 3)(x + 1) = 0 \text{ and } x = 3 \text{ or } x = -1$$

$$\text{When } x = 3, y = 1. \text{ When } x = -1, y = -3$$

Check

$$9 + 1 = 10 \text{ and } 1 + 9 = 10$$

$$(3, 1) \text{ and } (-1, -3)$$

9 Solve the simultaneous equations

$$y = x^2$$

$$x + y = 12$$

(5 marks)

Substitute $y = x^2$ into the linear equation: $x + x^2 = 12$

Rearrange

$$x^2 + x - 12 = 0$$

Factorise

$$(x + 4)(x - 3) = 0, \text{ so } x = -4 \text{ or } x = 3$$

When $x = 3$, $y = 9$. When $x = -4$, $y = 16$

Check in $x + y = 12$: $3 + 9 = 12$ and $-4 + 16 = 12$

$$x = 3, y = 9 \text{ and } x = -4, y = 16$$

10 Solve the simultaneous equations

$$y = x^2 - 2x$$

$$y = x + 1$$

Give your answers in the form $\frac{a \pm \sqrt{b}}{2}$ for x and for y , where a and b are integers.

(5 marks)

$$\begin{array}{rcl} x^2 - 2x & = & x + 1 \\ -x - 1 & & -x - 1 \\ \hline x^2 - 3x - 1 & = & 0 \end{array}$$

Does not factorise: use the formula with $a = 1$, $b = -3$, $c = -1$

$$x = \frac{3 \pm \sqrt{9 + 4}}{2}$$

$$x = \frac{3 \pm \sqrt{13}}{2}$$

Substitute into $y = x + 1$

$$y = \frac{3 \pm \sqrt{13}}{2} + 1$$

$$x = \frac{3 \pm \sqrt{13}}{2}, y = \frac{5 \pm \sqrt{13}}{2} \text{ (+ with +, - with -)}$$

11

The curve C has equation $y = x^2 + 3$. The line L has equation $y = 2x$.
Show that L does not intersect C .

(4 marks)

$$\begin{array}{r} x^2 + 3 = 2x \\ -2x \quad x^2 - 2x + 3 = 0 \quad -2x \end{array}$$

Discriminant $b^2 - 4ac$ with $a = 1$, $b = -2$, $c = 3$

$$\begin{aligned} b^2 - 4ac &= 4 - 12 \\ &= -8 \end{aligned}$$

Negative, so no real solutions

L does not intersect C

12

The curve C has equation $y = x^2 - 4x + 7$

The line L has equation $y = 2x - 2$

Show that L touches C at exactly one point, and find the coordinates of this point.

(4 marks)

Set the expressions for y equal

$$x^2 - 4x + 7 = 2x - 2$$

Rearrange

$$x^2 - 6x + 9 = 0$$

Factorise

$$(x - 3)^2 = 0$$

Both factors in $(x - 3)(x - 3)$ are zero at the same value, $x = 3$; this is called a repeated root. Also, curve minus line $= (x - 3)^2$ is never negative, so the curve stays on one side of the line and touches it at this one point

When $x = 3$, $y = 2(3) - 2 = 4$, so the point is $(3, 4)$

Check on the curve

$$9 - 12 + 7 = 4$$

13

Solve the simultaneous equations

$$x^2 + y^2 = 9$$

$$x + 2y = 3$$

(5 marks)

From the linear equation, $x = 3 - 2y$

Substitute

$$(3 - 2y)^2 + y^2 = 9$$

Expand

$$9 - 12y + 4y^2 + y^2 = 9, \text{ so } 5y^2 - 12y = 0$$

Factorise

$$y(5y - 12) = 0, \text{ so } y = 0 \text{ or } y = \frac{12}{5}$$

$$\text{When } y = 0, x = 3. \text{ When } y = \frac{12}{5}, x = 3 - \frac{24}{5} = -\frac{9}{5}$$

Check

$$9 + 0 = 9 \text{ and } \frac{81}{25} + \frac{144}{25} = \frac{225}{25} = 9$$

$$x = 3, y = 0 \text{ and } x = -\frac{9}{5}, y = \frac{12}{5}$$