



- 1** The point $(2, 5)$ lies on the graph of $y = f(x)$.

Write down the coordinates of the image of this point on each of these graphs.

a. $y = f(x + 3)$

b. $y = f(x) - 4$

c. $y = -f(x)$

d. $y = f(-x)$

(4 marks)

a) inside the bracket: x moves the opposite way

$(-1, 5)$

b) outside: y moves 4 down

$(2, 1)$

c) reflect in the x -axis

$(2, -5)$

d) reflect in the y -axis

$(-2, 5)$

- 2** The point $(6, -4)$ lies on the graph of $y = f(x)$.

Write down the coordinates of the image of this point on each of these graphs.

a. $y = 2f(x)$

b. $y = f(2x)$

c. $y = f\left(\frac{1}{3}x\right)$

d. $y = -3f(x)$

(4 marks)

a) multiply the y -coordinate by 2

$(6, -8)$

b) divide the x -coordinate by 2

$(3, -4)$

c) multiply the x -coordinate by 3

$(18, -4)$

d) multiply y by -3

$(6, 12)$

3 The point A(-2, 3) lies on the graph of $y = f(x)$.

Write down the coordinates of the image of A on each of these graphs.

a. $y = f(x - 3) + 1$

b. $y = -f(x + 1)$

c. $y = 2f(x) - 1$

d. $y = f(2x) + 5$

(4 marks)

a) 3 right, 1 up

$$= (-2 + 3, 3 + 1)$$

$$\boxed{(1, 4)}$$

b) 1 left, then $y \rightarrow -y$

$$= (-2 - 1, -3)$$

$$\boxed{(-3, -3)}$$

c) $y \rightarrow 2y - 1$

$$= (-2, 2 \times 3 - 1)$$

$$\boxed{(-2, 5)}$$

d) $x \rightarrow x \div 2$, then $y + 5$

$$= (-2 \div 2, 3 + 5)$$

$$\boxed{(-1, 8)}$$

4 The graph of $y = f(x)$ has a turning point at (3, -5).

a. Write down the coordinates of the turning point of the graph of $y = f(x) + 4$

b. Write down the coordinates of the turning point of the graph of $y = f(x - 2)$

(2 marks)

a) $y = f(x) + 4$ translates the graph 4 units up, so (3, -5) maps to $(3, -5 + 4) = (3, -1)$

b) $y = f(x - 2)$ translates the graph 2 units to the right, so (3, -5) maps to $(3 + 2, -5) = (5, -5)$

For part (b), the original turning point occurs when the input of f is 3. In $f(x - 2)$, solve $x - 2 = 3$, giving $x = 5$. This explains why subtracting inside the function moves the point to the right

5 The point A (4, 7) lies on the graph of $y = f(x)$.

a. Find the coordinates of the image of A on the graph of $y = f(-x)$

b. Find the coordinates of the image of A on the graph of $y = -f(x)$

(2 marks)

a) $y = f(-x)$ is a reflection in the y-axis: the x-coordinate changes sign, so A maps to (-4, 7)

b) $y = -f(x)$ is a reflection in the x-axis: the y-coordinate changes sign, so A maps to (4, -7)

6 The graph of $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$ has a maximum point at (90, 1).

a. Write down the coordinates of the maximum point of the graph of $y = \sin x - 3$

b. Write down the coordinates of the maximum point of the graph of $y = \sin(x - 30^\circ)$

(2 marks)

a) $y = \sin x - 3$ translates the graph 3 units down, so (90, 1) maps to (90, -2)

b) $y = \sin(x - 30)$ translates the graph 30 degrees to the right, so (90, 1) maps to (120, 1)

7 The graph of $y = f(x)$ passes through the point (2, 5).

a. Write down the coordinates of a point that must lie on the graph of $y = f(x + 6)$

b. Write down the coordinates of a point that must lie on the graph of $y = f(x) - 2$

(2 marks)

a) $y = f(x + 6)$ translates the graph 6 units to the left, so (2, 5) maps to (2 - 6, 5) = (-4, 5)

b) $y = f(x) - 2$ translates the graph 2 units down, so (2, 5) maps to (2, 3)

8 a. Describe fully the single transformation that maps the graph of $y = x^2$ onto the graph of $y = (x + 4)^2$

b. Describe fully the single transformation that maps the graph of $y = x^2$ onto the graph of $y = -x^2$

(2 marks)

a) Replacing x with $x + 4$ translates the graph 4 units in the negative x direction, a translation by the vector (-4, 0)

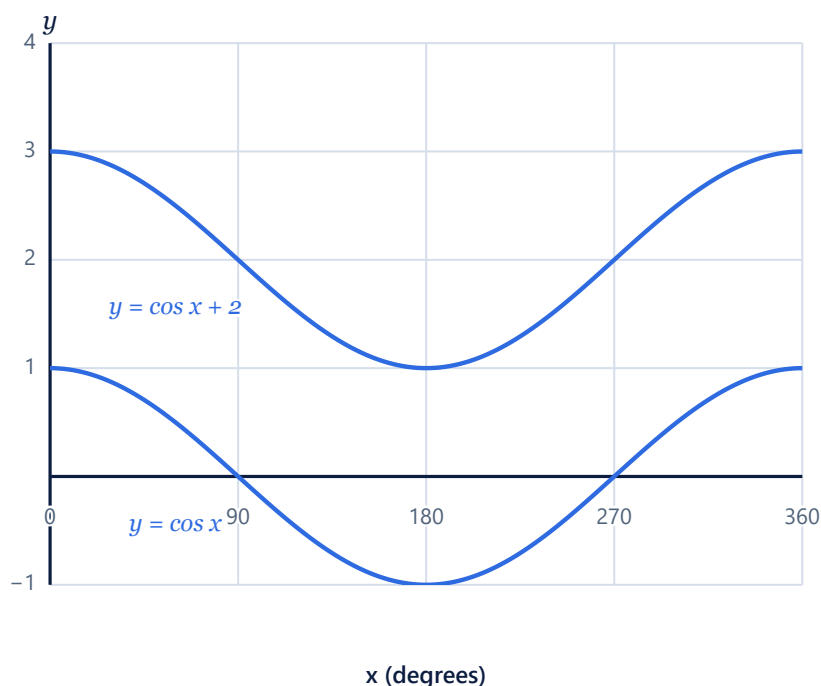
b) Multiplying the whole function by -1 reflects the graph in the x-axis

9

The graph of $y = \cos x$ is drawn for $0^\circ \leq x \leq 360^\circ$.

The graph of $y = \cos x + 2$ is sketched on the same axes for $0^\circ \leq x \leq 360^\circ$.

- Write down the coordinates of the minimum point of $y = \cos x + 2$
- Write down the coordinates of the maximum point of $y = \cos x + 2$ with the smallest x -coordinate



(2 marks)

$y = \cos x + 2$ translates the graph of $y = \cos x$ by 2 units up
 a) $y = \cos x$ has its minimum in this interval at $(180, -1)$, which maps to $(180, 1)$

b) $y = \cos x$ has maximum points at $(0, 1)$ and $(360, 1)$; the one with the smallest x -coordinate maps to $(0, 3)$

10

The curve C has equation $y = f(x)$ where $f(x) = x^2 + 2x$.

- Curve A is the reflection of C in the y -axis. Find the equation of curve A in the form $y = x^2 + bx + c$
- Curve B is the translation of C by 3 units in the positive y direction. Find the equation of curve B.

(3 marks)

a) A reflection in the y -axis replaces x with $-x$

$$f(-x) = (-x)^2 + 2(-x) = x^2 - 2x. \text{ So curve A is } y = x^2 - 2x$$

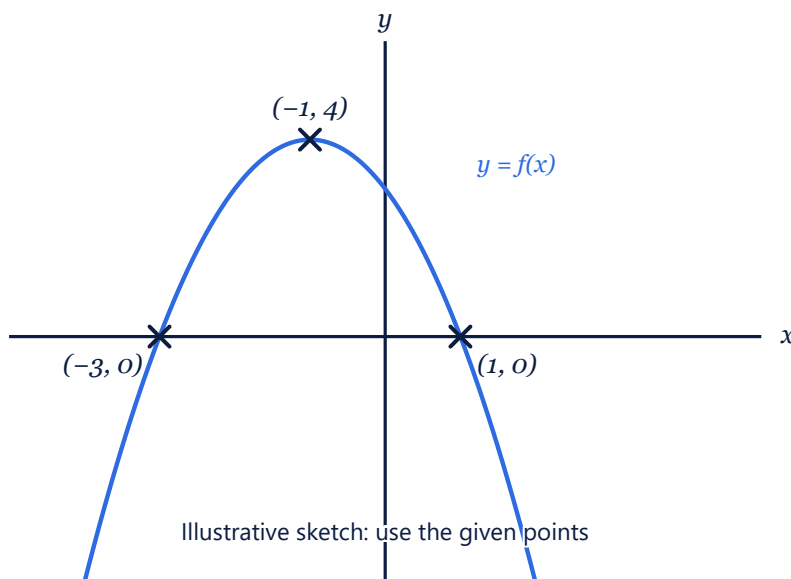
b) A translation 3 units up adds 3 to the function

$$y = f(x) + 3 = x^2 + 2x + 3$$

11

The graph of $y = f(x)$ has a maximum point at $(-1, 4)$ and crosses the x -axis at $(-3, 0)$ and $(1, 0)$.

- Write down the coordinates of the minimum point of the graph of $y = -f(x)$
- Write down the coordinates of the maximum point of the graph of $y = f(x - 2)$
- Write down the coordinates of the points where the graph of $y = f(x - 2)$ crosses the x -axis.



(3 marks)

- $y = -f(x)$ reflects the graph in the x -axis, so the maximum $(-1, 4)$ becomes a minimum at $(-1, -4)$
- $y = f(x - 2)$ translates the graph 2 units to the right, so the maximum moves to $(-1 + 2, 4) = (1, 4)$
- The x -intercepts also move 2 units right: $(-3, 0)$ maps to $(-1, 0)$ and $(1, 0)$ maps to $(3, 0)$

12

$$f(x) = x^2 - 4x + 7$$

- Write $f(x)$ in the form $(x - a)^2 + b$ and hence write down the coordinates of the turning point of $y = f(x)$
- Write down the coordinates of the turning point of $y = f(x - 3)$
- Find the equation of $y = f(-x)$, giving your answer in the form $y = x^2 + bx + c$

(4 marks)

a) Complete the square

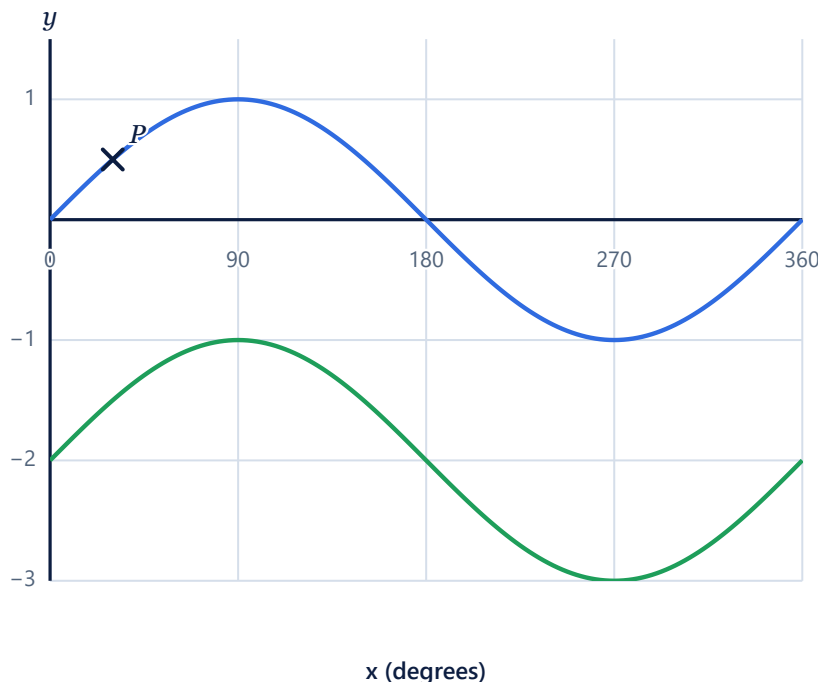
$$x^2 - 4x + 7 = (x - 2)^2 - 4 + 7 = (x - 2)^2 + 3. \text{ The turning point is } (2, 3)$$

b) $y = f(x - 3)$ translates the graph 3 units to the right, so the turning point moves to $(5, 3)$

$$c) f(-x) = (-x)^2 - 4(-x) + 7 = x^2 + 4x + 7, \text{ so } y = x^2 + 4x + 7$$

13 The diagram shows the graph of $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$.

- On the same axes, sketch the graph of $y = \sin x - 2$ for $0^\circ \leq x \leq 360^\circ$
- The point P (30, 0.5) lies on the graph of $y = \sin x$. P is mapped to the point Q on the graph of $y = \sin(x - 60^\circ)$. Find the coordinates of Q.



(4 marks)

- a) $y = \sin x - 2$ is a translation of $y = \sin x$ by 2 units down. The sketch passes through (0, -2), has a maximum at (90, -1), passes through (180, -2), has a minimum at (270, -3) and ends at (360, -2)
- b) $y = \sin(x - 60)$ translates the graph 60 degrees to the right, so P (30, 0.5) maps to Q (30 + 60, 0.5) = (90, 0.5)

14 $f(x) = \cos x^\circ$ for all real values of x

- Write down the coordinates of the minimum point of the graph of $y = f(x) + 2$
 - Write down the coordinates of the maximum point of the graph of $y = f(x - 30)$ for $0 \leq x \leq 360$
 - The graph of $y = f(x)$ is reflected in the x -axis. Write down the equation of the reflected graph.
- (4 marks)

- a) $y = f(x) + 2$ translates the curve up 2, so the minimum (180, -1) moves to (180, 1)
- b) $y = f(x - 30)$ translates the curve 30 to the right, so the maximum (0, 1) moves to (30, 1)
- c) Reflecting in the x -axis replaces y with $-y$, giving $y = -\cos x$

15

Describe fully the single transformation that maps the graph of $y = f(x)$ onto the graph of

- a. $y = f(x) + 5$
- b. $y = f(x - 4)$
- c. $y = -f(x)$
- d. $y = f(2x)$
- e. $y = 3f(x)$

(5 marks)

a) +5 outside: up

Translation by (0, 5)

b) -4 inside: moves right

Translation by (4, 0)

c) minus outside

Reflection in the x-axis

d) 2x inside: x-coordinates halve

Stretch, sf $1/2$, parallel to x-axise) 3 outside: y-coordinates $\times 3$

Stretch, sf 3, parallel to y-axis

16

The graph of $y = \tan x^\circ$ passes through (45, 1) and has an asymptote at $x = 90$.The graph of $y = \tan x^\circ$ is reflected in the x-axis and then translated 30 to the right and 2 up. This gives the graph C with equation $y = -\tan(x - 30)^\circ + 2$.

- a. Write down the coordinates of the image of (45, 1) on C.
- b. Write down the equation of the image of the asymptote $x = 90$ on C.
- c. Write down the y-coordinate of the point on C where $x = 30$.

(4 marks)

a)

$$= (45, 1) \rightarrow (45, -1)$$

$$= \rightarrow (45 + 30, -1 + 2)$$

$$(75, 1)$$

b) only the 30 right matters

$$90 + 30 = 120$$

$$x = 120$$

c)

$$y = -\tan(30 - 30)^\circ + 2 = -\tan 0^\circ + 2$$

$$y = 2$$

17

$$f(x) = x^2 - 4x$$

a. Write $f(x + 1)$ in the form $x^2 + ax + b$.

b. The graph of $y = f(x)$ has its minimum point at $(2, -4)$. Write down the minimum point of $y = f(x + 1)$.

(4 marks)

a)

$$\begin{aligned} f(x+1) &= (x+1)^2 - 4(x+1) \\ &= x^2 + 2x + 1 - 4x - 4 \end{aligned}$$

$$x^2 - 2x - 3$$

b) translation 1 to the left

$$(1, -4)$$