



- 1 OAB is a triangle.  
Vector  $OA = \mathbf{a}$  and vector  $OB = \mathbf{b}$ .  
M is the midpoint of OA and N is the midpoint of OB.  
Prove that MN is parallel to AB.

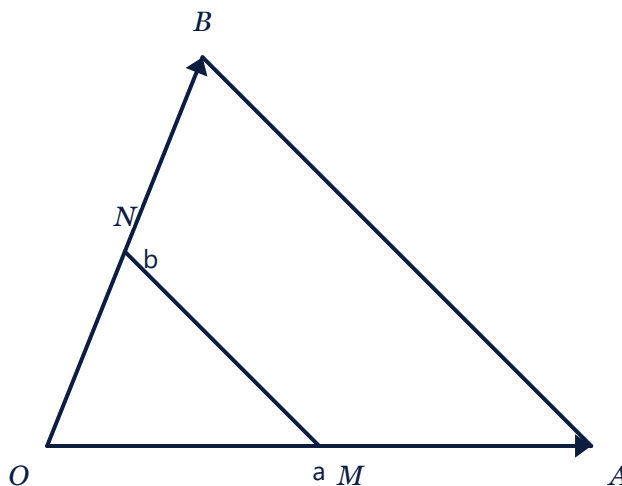


Diagram NOT accurately drawn

(4 marks)

$$\text{Vector } OM = \frac{1}{2}\mathbf{a} \text{ because M is the midpoint of OA}$$

$$\text{Vector } ON = \frac{1}{2}\mathbf{b} \text{ because N is the midpoint of OB}$$

$$\text{Vector } MN = ON - OM = \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a} = \frac{1}{2}(\mathbf{b} - \mathbf{a})$$

$$\text{Vector } AB = OB - OA = \mathbf{b} - \mathbf{a}$$

A scalar is an ordinary number. Multiplying a nonzero vector by a scalar changes its length (and may reverse it), but keeps its direction parallel

$$\text{So } MN = \frac{1}{2}AB, \text{ which means MN is a scalar multiple of AB}$$

Therefore MN is parallel to AB

2

ABCD is a quadrilateral.

Vector  $AB = 4\mathbf{a} + 2\mathbf{b}$

Vector  $BC = \mathbf{a} - 3\mathbf{b}$

Vector  $CD = -4\mathbf{a} - 2\mathbf{b}$

Prove that ABCD is a parallelogram.

(4 marks)

$$\text{Vector } DC = -CD = -(-4\mathbf{a} - 2\mathbf{b}) = 4\mathbf{a} + 2\mathbf{b}$$

$$\text{So } DC = AB$$

This means the sides AB and DC are parallel and equal in length

A quadrilateral with one pair of opposite sides that are equal and parallel is a parallelogram

Therefore ABCD is a parallelogram

Check with the remaining side

$$AD = AB + BC + CD = (4\mathbf{a} + 2\mathbf{b}) + (\mathbf{a} - 3\mathbf{b}) + (-4\mathbf{a} - 2\mathbf{b}) = \mathbf{a} - 3\mathbf{b} = BC, \text{ which confirms AD is equal and parallel to BC as expected}$$

3

Vector  $EF = 4\mathbf{a} - 2\mathbf{b}$

Vector  $GH = 6\mathbf{a} - 3\mathbf{b}$

Prove that EF is parallel to GH.

(4 marks)

Write both vectors with a common factor taken out

$$EF = 4\mathbf{a} - 2\mathbf{b} = 2(2\mathbf{a} - \mathbf{b})$$

$$GH = 6\mathbf{a} - 3\mathbf{b} = 3(2\mathbf{a} - \mathbf{b})$$

$$\text{So } GH = \frac{3}{2} \times EF, \text{ which means GH is a scalar multiple of EF}$$

Therefore EF is parallel to GH

4

P, Q and R are three points.

Vector  $PQ = 2\mathbf{a} - 3\mathbf{b}$

Vector  $QR = 6\mathbf{a} - 9\mathbf{b}$

Prove that P, Q and R lie on the same straight line.

(4 marks)

$$QR = 6\mathbf{a} - 9\mathbf{b} = 3(2\mathbf{a} - 3\mathbf{b}) = 3 \times PQ$$

So  $QR$  is a scalar multiple of  $PQ$ , which means  $PQ$  and  $QR$  are parallel

$PQ$  and  $QR$  both pass through the point  $Q$

Two parallel line segments through the same point lie on one straight line

Therefore P, Q and R lie on the same straight line (they are collinear)

5

OAB is a triangle.

Vector  $OA = 2\mathbf{a}$  and vector  $OB = 3\mathbf{b}$ .

M is the midpoint of OA.

N is the point on OB such that  $ON : NB = 2 : 1$ .

a. Find vector MN in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

b. Show that MN is not parallel to AB.

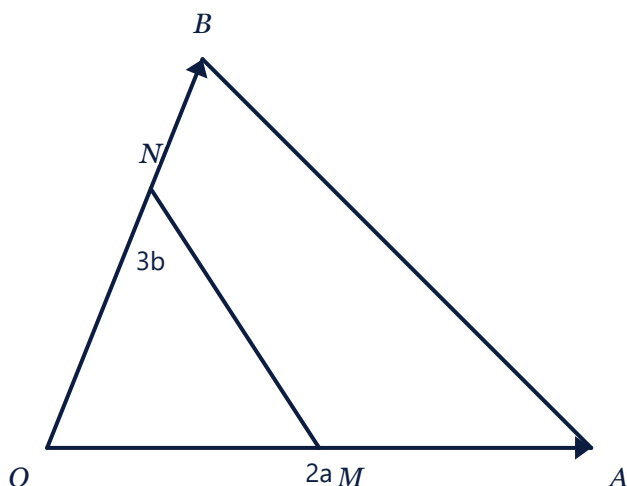


Diagram NOT accurately drawn

(5 marks)

a) Vector  $OM = \frac{1}{2} \times 2\mathbf{a} = \mathbf{a}$  because M is the midpoint of OA

Vector  $ON = \frac{2}{3} \times 3\mathbf{b} = 2\mathbf{b}$  because  $ON : NB = 2 : 1$

Vector  $MN = ON - OM = 2\mathbf{b} - \mathbf{a}$

b) Vector  $AB = OB - OA = 3\mathbf{b} - 2\mathbf{a}$

Suppose MN were parallel to AB. Then  $2\mathbf{b} - \mathbf{a} = k(3\mathbf{b} - 2\mathbf{a})$  for some scalar  $k$

Comparing the  $\mathbf{a}$  terms

$$-1 = -2k, \text{ so } k = \frac{1}{2}$$

Comparing the  $\mathbf{b}$  terms

$$2 = 3k, \text{ so } k = \frac{2}{3}$$

The two values of  $k$  are different, so no such scalar exists

Therefore MN is not parallel to AB

6

OACB is a parallelogram.

Vector  $\mathbf{OA} = \mathbf{a}$  and vector  $\mathbf{OB} = \mathbf{b}$ , so vector  $\mathbf{OC} = \mathbf{a} + \mathbf{b}$ .

M is the midpoint of OB.

X is the midpoint of OC.

D is the midpoint of AC.

Prove that M, X and D lie on the same straight line.

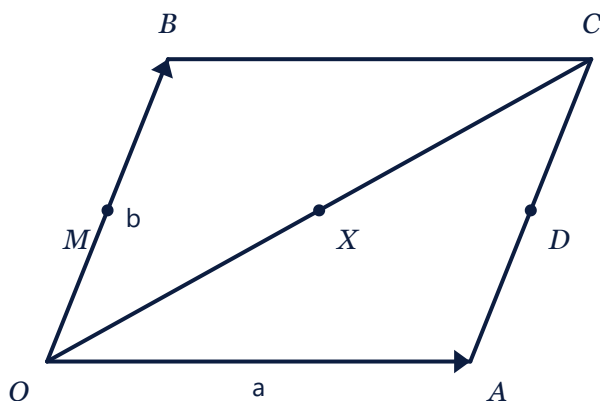


Diagram NOT accurately drawn

(5 marks)

$$\text{Vector } \mathbf{OM} = \frac{1}{2}\mathbf{b} \text{ because M is the midpoint of OB}$$

$$\text{Vector } \mathbf{OX} = \frac{1}{2}(\mathbf{a} + \mathbf{b}) \text{ because X is the midpoint of OC}$$

$$\begin{aligned} \text{O is the midpoint of AC, so vector } \mathbf{OD} &= \mathbf{OA} + \frac{1}{2}\mathbf{AC}. \text{ Vector } \mathbf{AC} = \mathbf{OC} - \mathbf{OA} = \mathbf{b}, \text{ so } \mathbf{OD} \\ &= \mathbf{a} + \frac{1}{2}\mathbf{b} \end{aligned}$$

$$\text{Vector } \mathbf{MX} = \mathbf{OX} - \mathbf{OM} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{b} = \frac{1}{2}\mathbf{a}$$

$$\text{Vector } \mathbf{XD} = \mathbf{OD} - \mathbf{OX} = \mathbf{a} + \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b} = \frac{1}{2}\mathbf{a}$$

So  $\mathbf{MX} = \mathbf{XD}$ . The two segments are parallel (both are scalar multiples of  $\mathbf{a}$ ) and share the point X

Therefore M, X and D lie on the same straight line, and X is the midpoint of MD

7

The points A and B have position vectors

$$OA = 4\mathbf{a} + 3\mathbf{b} \text{ and } OB = 8\mathbf{a} + 11\mathbf{b}.$$

- Find vector AB in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .
- The point P lies on AB such that  $AP : PB = 3 : 1$ . Work out vector OP in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .
- The point D has position vector  $OD = 14\mathbf{a} + 18\mathbf{b}$ . Prove that O, P and D lie on the same straight line.

(5 marks)

$$a) \text{ vector } AB = OB - OA = (8\mathbf{a} + 11\mathbf{b}) - (4\mathbf{a} + 3\mathbf{b}) = 4\mathbf{a} + 8\mathbf{b}$$

b) AP

$$PB = 3 : 1, \text{ so } AP = \frac{3}{4}AB = \frac{3}{4}(4\mathbf{a} + 8\mathbf{b}) = 3\mathbf{a} + 6\mathbf{b}$$

$$\text{Vector } OP = OA + AP = 4\mathbf{a} + 3\mathbf{b} + 3\mathbf{a} + 6\mathbf{b} = 7\mathbf{a} + 9\mathbf{b}$$

$$c) OD = 14\mathbf{a} + 18\mathbf{b} = 2(7\mathbf{a} + 9\mathbf{b}) = 2 \times OP$$

So OD is a scalar multiple of OP, which means OD and OP are parallel

OD and OP both pass through the point O

Therefore O, P and D lie on the same straight line, with P the midpoint of OD

8

The vectors **a** and **b** are non-parallel.

The points A, B and C have position vectors

$$OA = 2\mathbf{a} + 5\mathbf{b}$$

$$OB = 4\mathbf{a} + \lambda\mathbf{b}$$

$$OC = 8\mathbf{a} + 23\mathbf{b}$$

where  $\lambda$  is a number.

A, B and C lie on the same straight line.

Find the value of  $\lambda$ .

(5 marks)

$$\text{vector } AB = OB - OA = (4\mathbf{a} + \mathbf{b}) - (2\mathbf{a} + 5\mathbf{b}) = 2\mathbf{a} + (-5)\mathbf{b}$$

$$\text{vector } AC = OC - OA = (8\mathbf{a} + 23\mathbf{b}) - (2\mathbf{a} + 5\mathbf{b}) = 6\mathbf{a} + 18\mathbf{b}$$

A, B and C are collinear, so AB must be a scalar multiple of AC:  $2\mathbf{a} + (-5)\mathbf{b} = k(6\mathbf{a} + 18\mathbf{b})$

Comparing the a terms

$$2 = 6k, \text{ so } k = \frac{1}{3}$$

Comparing the b terms

$$-5 = 18k = 18 \times \frac{1}{3} = 6$$

$$\text{So } \lambda = 11$$

Check

with  $\lambda = 11$ ,  $AB = 2\mathbf{a} + 6\mathbf{b}$  and  $AC = 6\mathbf{a} + 18\mathbf{b} = 3AB$ , so the three points are indeed collinear

11

9

OAB is a triangle.

Vector  $OA = \mathbf{a}$  and vector  $OB = \mathbf{b}$ .

M is the midpoint of AB.

N is the midpoint of OB.

P is the point on OM such that  $OP : PM = 2 : 1$ .

a. Prove that A, P and N lie on the same straight line.

b. Find  $AP : PN$ .

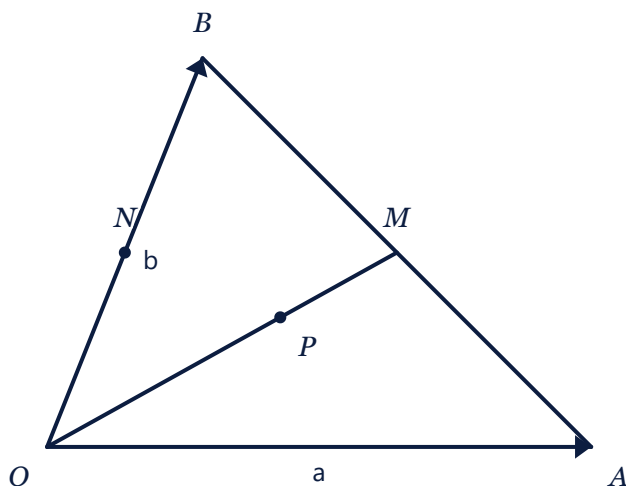


Diagram NOT accurately drawn

(5 marks)

a) Vector  $OM = \frac{1}{2}(\mathbf{a} + \mathbf{b})$  because M is the midpoint of AB

OP

$$PM = 2 : 1, \text{ so } OP = \frac{2}{3}OM = \frac{1}{3}(\mathbf{a} + \mathbf{b})$$

$$\text{Vector } AP = OP - OA = \frac{1}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} - \mathbf{a} = -\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$$

$$\text{Vector } ON = \frac{1}{2}\mathbf{b} \text{ because N is the midpoint of OB}$$

$$\text{Vector } AN = ON - OA = -\mathbf{a} + \frac{1}{2}\mathbf{b}$$

$$\frac{3}{2} \times AP = \frac{3}{2}\left(-\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}\right) = -\mathbf{a} + \frac{1}{2}\mathbf{b} = AN$$

So  $AN = \frac{3}{2}AP$ , which means AP and AN are parallel; both pass through A, so A, P and

N lie on the same straight line

$$\text{b) } PN = AN - AP = \frac{3}{2}AP - AP = \frac{1}{2}AP$$

So AP

$$PN = 1 : \frac{1}{2} = 2 : 1$$

This is the well-known result that the medians of a triangle meet at a point that divides each median in the ratio 2 : 1

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OAB is a triangle.

Vector  $OA = 3\mathbf{a}$  and vector  $OB = 3\mathbf{b}$ .

P is the point on AB such that  $AP : PB = 1 : 2$ .

- Find vector AB in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .
- Show that vector  $OP = 2\mathbf{a} + \mathbf{b}$ .
- The point Q lies on OP produced, so that BQ is parallel to OA. Work out  $OP : PQ$ .

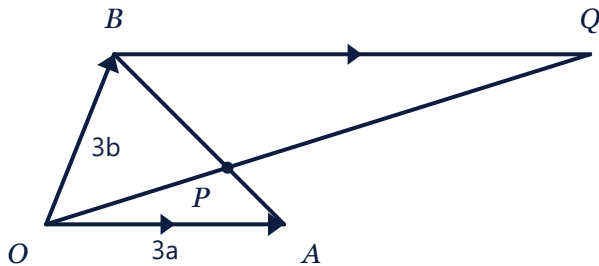


Diagram NOT accurately drawn

(6 marks)

$$a) \text{ vector } AB = OB - OA = 3b - 3a$$

b) AP

$$PB = 1 : 2, \text{ so } AP = \frac{1}{3}AB = \frac{1}{3}(3b - 3a) = b - a$$

$$\text{vector } OP = OA + AP = 3a + b - a = 2a + b, \text{ as required}$$

c) Q lies on OP produced, so  $OQ = t(2a + b)$  for some scalar  $t > 1$

$$\text{vector } BQ = OQ - OB = 2ta + tb - 3b = 2ta + (t - 3)b$$

BQ is parallel to  $OA = 3a$ , so BQ has no b component:  $t - 3 = 0$ , which gives  $t = 3$

So  $OQ = 3 \times OP = 6a + 3b$ , and check:  $BQ = 6a + 3b - 3b = 6a$ , which is parallel to OA

$$PQ = OQ - OP = 3OP - OP = 2OP$$

Therefore OP

$$PQ = 1 : 2$$

11

OAB is a triangle.  $OA = \mathbf{a}$  and  $OB = \mathbf{b}$ .

M is the midpoint of OA and N is the midpoint of OB.

Show that MN is parallel to AB.

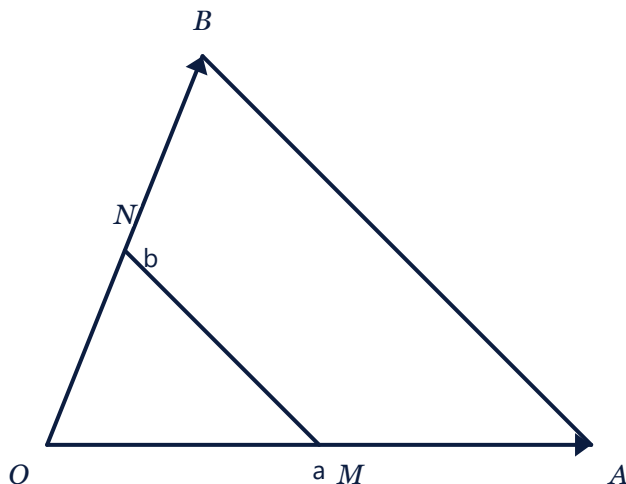


Diagram NOT accurately drawn

(3 marks)

$$OM = \frac{1}{2}\mathbf{a} \text{ and } ON = \frac{1}{2}\mathbf{b}$$

$$MN = ON - OM = \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a} = \frac{1}{2}(\mathbf{b} - \mathbf{a})$$

$$AB = OB - OA = \mathbf{b} - \mathbf{a}$$

$MN = \frac{1}{2}AB$ , so MN is a scalar multiple of AB and is therefore parallel to AB

A scalar is an ordinary number multiplying a vector. Multiplying a non-zero vector by  $1/2$  halves its length without changing its direction, which establishes parallel lines

12

OAB is a triangle.  $OA = \mathbf{a}$  and  $OB = \mathbf{b}$ .

P is the point on AB such that  $AP : PB = 2 : 1$ .

Express OP in terms of  $\mathbf{a}$  and  $\mathbf{b}$ . Give your answer in its simplest form.

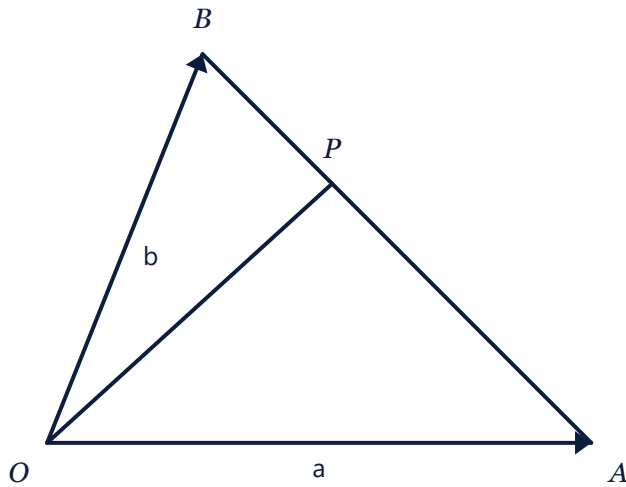


Diagram NOT accurately drawn

(4 marks)

$$AB = OB - OA = b - a$$

AP

$$PB = 2 : 1, \text{ so } AP = \left(\frac{2}{3}\right)AB = \left(\frac{2}{3}\right)(b - a)$$

$$OP = OA + AP = a + \left(\frac{2}{3}\right)(b - a)$$

$$OP = a - \left(\frac{2}{3}\right)a + \left(\frac{2}{3}\right)b = \left(\frac{1}{3}\right)a + \left(\frac{2}{3}\right)b$$

$$\boxed{\left(\frac{1}{3}\right)a + \left(\frac{2}{3}\right)b}$$

13

OABC is a trapezium.  $OA = \mathbf{a}$  and  $OC = \mathbf{c}$ .

CB is parallel to OA and  $CB = 2\mathbf{a}$ .

M is the midpoint of AB.

Find OM in terms of  $\mathbf{a}$  and  $\mathbf{c}$ . Give your answer in its simplest form.

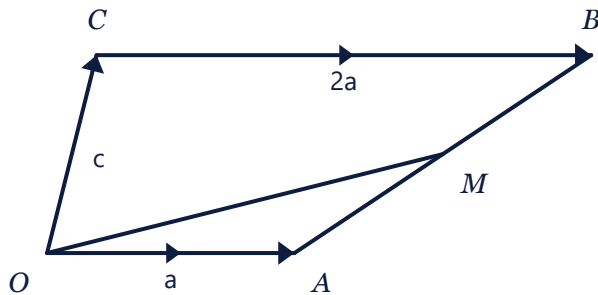


Diagram NOT accurately drawn

(4 marks)

$$OB = OC + CB = c + 2a$$

$$AB = OB - OA = c + 2a - a = a + c$$

$$AM = \frac{1}{2}AB = \frac{1}{2}(a + c)$$

$$OM = OA + AM = a + \frac{1}{2}a + \frac{1}{2}c = \left(\frac{3}{2}\right)a + \left(\frac{1}{2}\right)c$$

$$\boxed{\left(\frac{3}{2}\right)a + \left(\frac{1}{2}\right)c}$$