Calculus in kinematics

Worked solutions

GCSE Maths topic practice

This topic online

1 A particle moves along a straight line. At time t seconds its displacement, s metres, from a fixed point O is given by

$s = 3t^2 + 4t$

a. Find an expression for the velocity, v m/s, of the particle at time t seconds.

b. Find the velocity of the particle when $t = 2$.

(2 marks)

a) $v = \frac{ds}{dt}$

$v = 6t + 4$

$\boxed{v = 6t + 4}$

b) put $t = 2$ into v

$v = 6 \times 2 + 4$

$\boxed{16 \text{ m/s}}$

2 A particle moves along a straight line. At time t seconds its velocity, v m/s, is given by

$v = 12t - 2t^2$

a. Find an expression for the acceleration, a m/s^2, of the particle at time t seconds.

b. Find the acceleration of the particle when $t = 5$.

(2 marks)

a) $a = \frac{dv}{dt}$

$a = 12 - 4t$

$\boxed{a = 12 - 4t}$

b) put $t = 5$ into a

$a = 12 - 4 \times 5$

$= 12 - 20$

$\boxed{-8 \text{ m/s}^2}$

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3 A particle P moves along a straight line through a fixed point O. At time t seconds the displacement, s metres, of P from O is given by

$s = t^3 - 6t^2 + 9t + 2$

a. Find an expression for the velocity, v m/s, of P at time t seconds.

b. Find an expression for the acceleration, a m/s^2, of P at time t seconds.

c. Find the acceleration of P when $t = 4$.

(4 marks)

a) $v = \frac{ds}{dt}$, the + 2 vanishes

$v = 3t^2 - 12t + 9$

$\boxed{v = 3t^2 - 12t + 9}$

b) $a = \frac{dv}{dt}$

$a = 6t - 12$

$\boxed{a = 6t - 12}$

c) put $t = 4$ into a

$a = 6 \times 4 - 12$

$\boxed{12 \text{ m/s}^2}$

4 A particle moves along a straight line. At time t seconds its displacement, s metres, from a fixed point O is given by

$s = 2t^3 - 15t^2 + 24t$ for $0 \le t \le 6$

Find the values of t for which the particle is momentarily at rest.

(3 marks)

At rest means $v = 0$

$v = \frac{ds}{dt} = 6t^2 - 30t + 24$

$6t^2 - 30t + 24 = 0$

$\div 6$ $\quad t^2 - 5t + 4 = 0$ $\quad \div 6$

$(t - 1)(t - 4) = 0$

$\boxed{t = 1 \text{ or } t = 4}$

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5 A particle P moves along a straight line through a fixed point O. At time t seconds the displacement, s metres, of P from O is given by

$s = t^3 - 9t^2 + 5t$ for $t \geq 0$

a. Find an expression for the acceleration, a m/s^2, of P at time t seconds.

b. Find the value of t when the acceleration of P is zero. (3 marks)

a) differentiate twice

$$v = \frac{ds}{dt} = 3t^2 - 18t + 5$$

$$a = \frac{dv}{dt} = 6t - 18$$

$a = 6t - 18$

b) set $a = 0$

$6t - 18 = 0$

$+18$ $6t = 18$ $+18$

$\div 6$ $t = 3$ $\div 6$

$t = 3$

6 The number of bacteria, N, in a dish t hours after the start of an experiment is given by

$N = 400 + 30t^2 - t^3$ for $0 \leq t \leq 20$

a. Find an expression for $\frac{dN}{dt}$.

b. Find the rate at which the number of bacteria is increasing, in bacteria per hour, when $t = 2$. (3 marks)

a) rate of change $= \frac{dN}{dt}$, the 400 vanishes

$$\frac{dN}{dt} = 60t - 3t^2$$

$60t - 3t^2$

b) put $t = 2$ in

$$\frac{dN}{dt} = 60 \times 2 - 3 \times 2^2$$

$$= 120 - 12$$

108 bacteria per hour

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7 Priya throws a ball straight up into the air. The height, h metres, of the ball above the ground t seconds after it leaves her hand is given by

$h = 30t - 5t^2$

a. Find the value of t when the ball reaches its greatest height.

b. Find the greatest height, in metres, that the ball reaches.

(4 marks)

a) at the top the velocity is 0

$$\frac{dh}{dt} = 30 - 10t$$

$$30 - 10t = 0$$

$+10t$ $\quad 10t = 30 \quad +10t$

$\div 10$ $\quad t = 3 \quad \div 10$

$\boxed{t = 3}$

b) $-5t^2$ means a maximum, put $t = 3$ into h

$h = 30 \times 3 - 5 \times 3^2$

$= 90 - 45$

$\boxed{45 \text{ m}}$

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8 A train moves along a straight track. At time t seconds its velocity, v m/s, is given by

$v = 24t - 3t^2$ for $0 \leq t \leq 8$

a. Find an expression for the acceleration, a m/s^2, of the train at time t seconds.

b. Find the value of t when the velocity of the train is greatest.

c. Find the greatest velocity of the train, in m/s.

(4 marks)

a) $a = \frac{dv}{dt}$

$a = 24 - 6t$

$\boxed{a = 24 - 6t}$

b) greatest v where a = 0

$24 - 6t = 0$

$+6t$ $\quad 6t = 24$ $\quad +6t$

$\div 6$ $\quad t = 4$ $\quad \div 6$

$-3t^2$ so the graph of v is n-shaped: a maximum

$\boxed{t = 4}$

c) put t = 4 into v

$v = 24 \times 4 - 3 \times 4^2$

$= 96 - 48$

$\boxed{48 \text{ m/s}}$

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9 A particle P moves along a straight line. At time t seconds the displacement, s metres, of P from a fixed point O is given by

$s = 6t^2 - t^3$

P starts at O when $t = 0$ and later returns to O.

a. Find the value of t when P returns to O.

b. Find the velocity of P, in m/s, at the instant it returns to O. (4 marks)

a) at O means $s = 0$

$6t^2 - t^3 = 0$

$t^2(6 - t) = 0$

$t = 0$ is the start, so take the other root

$\boxed{t = 6}$

b) $v = \frac{ds}{dt}$, then put $t = 6$ in

$v = 12t - 3t^2$

$v = 12 \times 6 - 3 \times 6^2$

$= 72 - 108$

$\boxed{-36 \text{ m/s}}$

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10 A particle moves along a straight line through a fixed point O. At time t seconds the displacement, s metres, of the particle from O is given by

$s = 2t^3 - 21t^2 + 60t$ for $0 \le t \le 6$

a. Find the values of t for which the velocity of the particle is zero.

b. Find the greatest displacement, in metres, of the particle from O during the 6 seconds. (5 marks)

a) $v = \frac{ds}{dt} = 0$

$v = 6t^2 - 42t + 60$

$6t^2 - 42t + 60 = 0$

$\div 6$ $t^2 - 7t + 10 = 0$ $\div 6$

$(t - 2)(t - 5) = 0$

$t = 2$ or $t = 5$

b) positive t^3 cubic: max first ($t = 2$), min second ($t = 5$)

$s(2) = 2 \times 8 - 21 \times 4 + 60 \times 2$

$= 16 - 84 + 120 = 52$

check the end of the interval too

$s(6) = 2 \times 216 - 21 \times 36 + 60 \times 6$

$= 432 - 756 + 360 = 36$

$36 < 52$, and $s(0) = 0$

52 m (at $t = 2$)

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