



- 1 A circle has equation $(x - 2)^2 + (y + 1)^2 = 25$.

The point P(5, 3) lies on the circle.

Find an equation of the tangent to the circle at P. Give your answer in the form $ax + by = c$.

(4 marks)

centre = (2, -1)

$$\begin{aligned} \text{gradient of radius} &= \{3 - (-1)\} / \{5 - 2\} = \{4\} / \{3\} \\ \text{gradient of tangent} &= -\{3\} / \{4\} \\ y - 3 &= -\{3\} / \{4\} (x - 5) \\ 4y - 12 &= -3x + 15 \\ 3x + 4y &= 27 \end{aligned}$$

$$\boxed{3x + 4y = 27}$$

2

The circle C has equation $x^2 + y^2 = 25$

The point P (3, 4) lies on C.

Find an equation of the tangent to C at the point P.

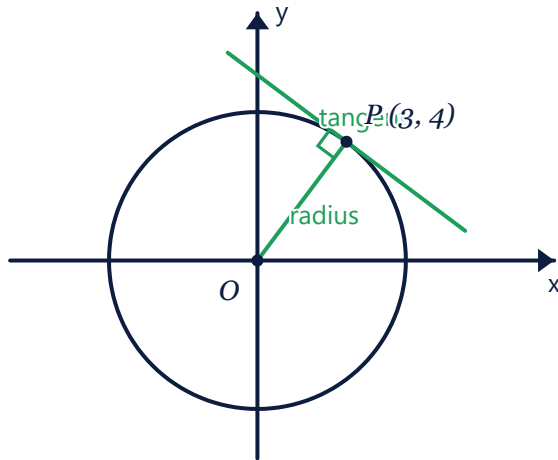


Diagram NOT accurately drawn

(4 marks)

Check P lies on C

$$3^2 + 4^2 = 9 + 16 = 25$$

O is the origin (0, 0). The radius gradient is the change in y divided by the change in x from O to P

The gradient of the radius OP is $\frac{4}{3}$

The tangent is perpendicular to the radius at P, so its gradient is $-\frac{3}{4}$ (since $\frac{4}{3}$ times $-\frac{3}{4} = -1$)

Using $y - 4 = -\frac{3}{4}(x - 3)$:

$$y = -\frac{3x}{4} + \frac{9}{4} + 4 = -\frac{3x}{4} + \frac{25}{4}, \text{ which can be written } 3x + 4y = 25$$

$$\boxed{y = -\frac{3x}{4} + \frac{25}{4}}$$

3

The circle C has equation $x^2 + y^2 = 29$

The point P $(-2, 5)$ lies on C.

Find an equation of the tangent to C at the point P.

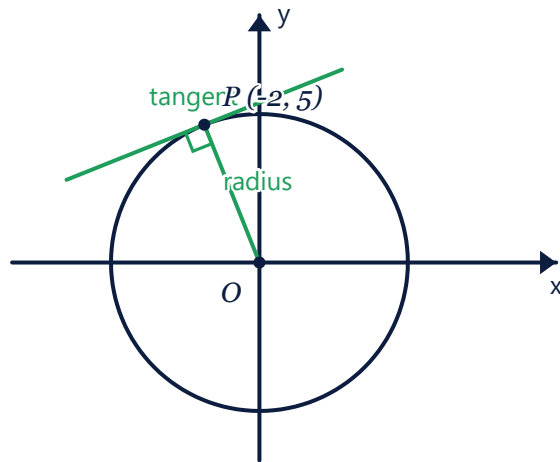


Diagram NOT accurately drawn

(4 marks)

Check P lies on C

$$(-2)^2 + 5^2 = 4 + 25 = 29$$

O is the origin $(0, 0)$. The radius gradient is the change in y divided by the change in x from O to P

The gradient of the radius OP is $\frac{5}{-2} = -\frac{5}{2}$

The tangent is perpendicular to the radius, so its gradient is $\frac{2}{5}$ (since $-\frac{5}{2}$ times $\frac{2}{5} = -1$)

Using $y - 5 = \frac{2}{5}(x + 2)$:

$$y = \frac{2x}{5} + \frac{4}{5} + 5 = \frac{2x}{5} + \frac{29}{5}, \text{ which can be written } 5y = 2x + 29$$

$$\boxed{y = \frac{2x}{5} + \frac{29}{5}}$$

- 4 A circle has equation $(x + 3)^2 + (y - 2)^2 = 13$.

The point $P(-1, 5)$ lies on the circle.

Find an equation of the tangent to the circle at P . Give your answer in the form $ax + by = c$.

(4 marks)

centre = $(-3, 2)$

$$\text{gradient of radius} = \frac{5 - 2}{-1 + 3} = \frac{3}{2}$$

$$\text{gradient of tangent} = -\frac{2}{3}$$

$$y - 5 = -\frac{2}{3}(x + 1)$$

$\times 3$

$$3y - 15 = -2x - 2$$

$\times 3$

$$2x + 3y = 13$$

$2x + 3y = 13$

- 5 A circle has centre O , the origin, and passes through the point $T(5, 12)$.

a. Find an equation of the circle.

b. The point D has coordinates $(10, 9)$. Decide whether D is inside the circle, on the circle, or outside the circle. You must show your working.

(4 marks)

a) The radius squared is $OT^2 = 5^2 + 12^2 = 25 + 144 = 169$ (radius 13)

So the circle has equation $x^2 + y^2 = 169$

b) For D

$$10^2 + 9^2 = 100 + 81 = 181$$

$181 > 169$, so D is further from O than the radius: D is outside the circle

6

The circle C has centre O , the origin, and equation $x^2 + y^2 = 100$

The tangent to C at the point $P(6, 8)$ crosses the x -axis at the point A .

Find the exact coordinates of A .

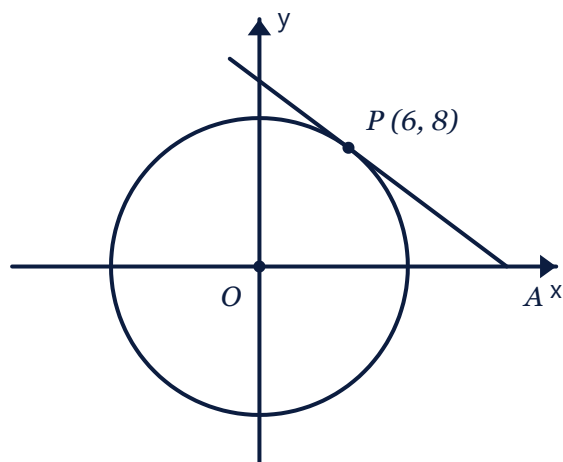


Diagram NOT accurately drawn

(4 marks)

Check P lies on C

$$6^2 + 8^2 = 36 + 64 = 100$$

The gradient of the radius OP is $\frac{8}{6} = \frac{4}{3}$, so the tangent at P has gradient $-\frac{3}{4}$

Tangent

$$y - 8 = -\frac{3}{4}(x - 6), \text{ so } y = -\frac{3x}{4} + \frac{9}{2} + 8 = -\frac{3x}{4} + \frac{25}{2}$$

$$\text{At } A, y = 0: \frac{3x}{4} = \frac{25}{2}, \text{ so } x = \frac{50}{3}$$

A is the point $(\frac{50}{3}, 0)$

$$\boxed{(\frac{50}{3}, 0)}$$

7 The circle C has centre O, the origin, and equation $x^2 + y^2 = 45$

A tangent to C touches the circle at the point P.

The tangent has gradient $-\frac{1}{2}$ and P is in the first quadrant.

Find the coordinates of P.

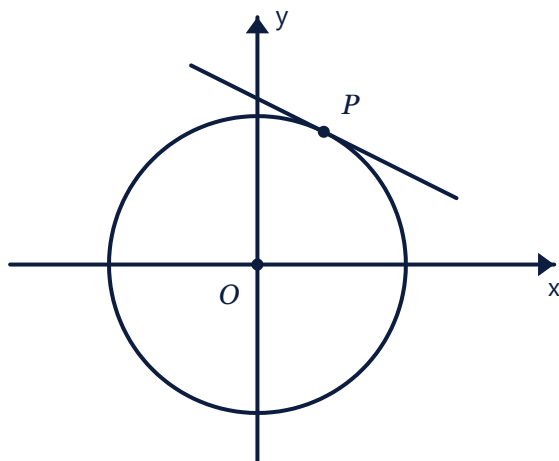


Diagram NOT accurately drawn

(4 marks)

The radius OP is perpendicular to the tangent, so the gradient of OP is 2 (the negative reciprocal of $-\frac{1}{2}$)

P lies on the line $y = 2x$ through the origin

Substitute into the circle equation

$$x^2 + (2x)^2 = 45, \text{ so } 5x^2 = 45 \text{ and } x^2 = 9$$

$$x = 3 \text{ or } x = -3. P \text{ is in the first quadrant, so } x = 3 \\ \text{and } y = 6$$

Check

$$3^2 + 6^2 = 9 + 36 = 45, \text{ so } P(3, 6) \text{ lies on } C$$

$(3, 6)$

8

A circle has equation $x^2 + y^2 = 20$.

The point A(4, 2) lies on the circle.

- Find an equation of the tangent to the circle at A.
- The tangent meets the x -axis at P and the y -axis at Q. Work out the area of triangle OPQ, where O is the origin.

(5 marks)

a)

$$\text{gradient of OA} = \frac{2}{4} = \frac{1}{2}$$

$$\text{gradient of tangent} = -2$$

$$y - 2 = -2(x - 4)$$

$$y = -2x + 10$$

b)

$$y = 0: 0 = -2x + 10, x = 5$$

$$x = 0: y = 10$$

$$P = (5, 0), Q = (0, 10)$$

$$\text{area} = \frac{1}{2} \times 5 \times 10$$

$$\boxed{\text{a) } y = -2x + 10 \text{ b) } 25}$$

9

The line with equation $3x + 4y = 30$ is a tangent to a circle with centre O, the origin.
Find an equation of the circle.

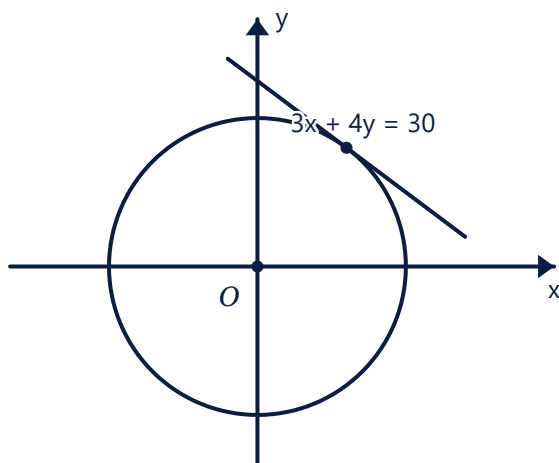


Diagram NOT accurately drawn

(5 marks)

The radius to the point of contact is perpendicular to the tangent

The tangent $3x + 4y = 30$ has gradient $-\frac{3}{4}$, so the radius lies on the line $y = \frac{4x}{3}$ through O

Substitute $y = \frac{4x}{3}$ into $3x + 4y = 30$: $3x + \frac{16x}{3} = 30$, so $\frac{25x}{3} = 30$ and $x = \frac{18}{5}$

Then $y = \frac{4}{3}$ times $\frac{18}{5} = \frac{24}{5}$. The point of contact is $(\frac{18}{5}, \frac{24}{5})$

$$r^2 = (\frac{18}{5})^2 + (\frac{24}{5})^2 = \frac{324}{25} + \frac{576}{25} = \frac{900}{25} = 36$$

So the circle has equation $x^2 + y^2 = 36$ (radius 6, which matches the distance $\frac{30}{5} = 6$ from O to the line)

$x^2 + y^2 = 36$

10

The centre of a circle is $C(1, 2)$. The point $P(4, -2)$ lies on the circle.

a. Work out the radius of the circle.

b. Find an equation of the tangent to the circle at P . Give your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(5 marks)

a)

$$r^2 = 3^2 + 4^2 = 25$$

$$r = 5$$

b)

$$\text{gradient of } CP = \frac{-2 - 2}{4 - 1} = -\frac{4}{3}$$

$$\text{gradient of tangent} = \frac{3}{4}$$

$$y + 2 = \frac{3}{4}(x - 4)$$

$\times 4$

$$4y + 8 = 3x - 12$$

$\times 4$

$$3x - 4y - 20 = 0$$

$$\boxed{a) 5 \quad b) 3x - 4y - 20 = 0}$$