



1 $f(x) = 2x + 1$ and $g(x) = x - 4$

Find $(fg)^{-1}(x)$.

(3 marks)

$$\begin{aligned}
 fg(x) &= 2(x - 4) + 1 = 2x - 7 \\
 y &= 2x - 7 \\
 +7 \quad y + 7 &= 2x \quad +7 \\
 x &= \frac{y + 7}{2} \\
 \boxed{(fg)^{-1}(x) = \frac{x + 7}{2}}
 \end{aligned}$$

2 $f(x) = 3x - 2$ and $g(x) = x + 5$

Show that $(fg)^{-1}(x) = g^{-1}f^{-1}(x)$.

(4 marks)

Left-hand side

$$\begin{aligned}
 fg(x) &= 3(x + 5) - 2 = 3x + 13 \\
 (fg)^{-1}(x) &= \frac{x - 13}{3}
 \end{aligned}$$

Right-hand side

$$\begin{aligned}
 f^{-1}(x) &= \frac{x + 2}{3} \\
 g^{-1}(x) &= x - 5 \\
 g^{-1}f^{-1}(x) &= \frac{x + 2}{3} - 5 \\
 &= \frac{x + 2 - 15}{3}
 \end{aligned}$$

$$\boxed{\text{Both} = \frac{x - 13}{3}}$$

3

$$f(x) = x + 3 \quad g(x) = 4x$$

- Find $fg(x)$
- Find $(fg)^{-1}(x)$
- Find $(fg)^{-1}(11)$

(4 marks)

$$a) fg(x) = f(4x) = 4x + 3$$

$$b) \text{ Write } y = 4x + 3 \text{ and make } x \text{ the subject: } x = \frac{y - 3}{4}, \text{ so}$$

$$(fg)^{-1}(x) = \frac{x - 3}{4}$$

$$c) (fg)^{-1}(11) = \frac{11 - 3}{4} = 2. \text{ Check: } fg(2) = 8 + 3 = 11$$

4

$$f(x) = 5x - 3 \quad g(x) = \frac{x + 1}{2}$$

- Find $gf(1)$
- Find $g^{-1}f(2)$

(4 marks)

$$a) f(1) = 5 - 3 = 2, \text{ then } g(2) = \frac{2 + 1}{2} = \frac{3}{2}$$

$$b) g^{-1}(x) = 2x - 1, \text{ since } y = \frac{x + 1}{2} \text{ gives } x = 2y - 1$$

$$b) f(2) = 10 - 3 = 7, \text{ then } g^{-1}(7) = 14 - 1 = 13$$

5

$$f(x) = \frac{2x+1}{x-3}, x \neq 3$$

a. Show that $f^{-1}(x) = \frac{3x+1}{x-2}$

b. $g(x) = x + 4$

Solve $fg(x) = 5$

(6 marks)

a) Write $y = \frac{2x+1}{x-3}$ and multiply up: $y(x-3) = 2x+1$

a) Expand and collect x terms

$$xy - 2x = 3y + 1, \text{ so } x(y-2) = 3y+1$$

a) Divide

$$x = \frac{3y+1}{y-2}, \text{ so } f^{-1}(x) = \frac{3x+1}{x-2} \text{ as required}$$

The inverse is defined for $x \neq 2$

f never outputs 2, since $(2x+1)/(x-3) = 2$ would imply $1 = -6$. In part (b), $x \neq -1$ because $g(x)$ must not equal 3

$$\begin{aligned} \text{b) } fg(x) &= f(x+4) = \\ &= \frac{2(x+4)+1}{(x+4)-3} = \\ &= \frac{2x+9}{x+1} \end{aligned}$$

$$\begin{aligned} \text{b) Solve } \frac{2x+9}{x+1} &= 5: 2x+9 = 5x+5, \text{ so } 3x = 4 \text{ and } x \\ &= \frac{4}{3} \end{aligned}$$

b) Check

$$g\left(\frac{4}{3}\right) = \frac{16}{3} \text{ and } f\left(\frac{16}{3}\right) = \frac{35}{3} \cdot \frac{7}{3} = 5$$

6

$$f(x) = x + 2 \quad g(x) = x^2 - 3x$$

- Find $fg(1)$
- Find $f^{-1}(x)$
- Solve $gf(x) = 10$

(7 marks)

$$a) g(1) = 1 - 3 = -2, \text{ then } f(-2) = 0$$

$$b) y = x + 2 \text{ gives } x = y - 2, \text{ so } f^{-1}(x) = x - 2$$

$$c) gf(x) = (x + 2)^2 - 3(x + 2) = x^2 + 4x + 4 - 3x - 6 = x^2 + x - 2$$

$$c) \text{ Solve } x^2 + x - 2 = 10, \text{ so } x^2 + x - 12 = 0$$

c) Factorise

$$(x + 4)(x - 3) = 0, \text{ so } x = -4 \text{ or } x = 3$$

$$c) \text{ Check } x = 3: f(3) = 5, g(5) = 25 - 15 = 10. \text{ Check } x = -4: f(-4) = -2, g(-2) = 4 - 6 = -2$$

7

$$f(x) = \frac{1}{x+2} \text{ for } x \neq -2 \text{ and } g(x) = 3x - 1$$

Find $(fg)^{-1}(x)$.

(4 marks)

$$\begin{array}{lcl} fg(x) & = & 3x - 1 + 2 = 3x + 1 \\ y & = & 3x + 1 \\ \times(3x+1) & y(3x+1) & = 1 \\ \div y & 3x+1 & = \frac{1}{y} \\ -1 & 3x & = \frac{1}{y} - 1 = \frac{1-y}{y} \\ \div 3 & x & = \frac{1-y}{3y} \end{array}$$

$$(fg)^{-1}(x) = \frac{1-x}{3x}$$

8 $f(x) = x^2 + 3$ for $x \geq 0$ and $g(x) = 2x - 1$ for $x \geq 1$.

Find $(fg)^{-1}(x)$.

(4 marks)

$$\begin{aligned}
 fg(x) &= (2x - 1)^2 + 3 \\
 y &= (2x - 1)^2 + 3 \\
 -3 \quad y - 3 &= (2x - 1)^2 \quad -3 \\
 \sqrt{y - 3} &= 2x - 1 \\
 \text{Positive root only, since } x \geq 1 \\
 +1 \quad 2x &= 1 + \sqrt{y - 3} \quad +1 \\
 x &= \frac{1 + \sqrt{y - 3}}{2} \\
 (fg)^{-1}(x) &= \frac{1 + \sqrt{x - 3}}{2}
 \end{aligned}$$

9 $f(x) = 3x - 4$ and $g(x) = x^2 - 2$

Solve $f^{-1}(x) = g(x)$.

(4 marks)

$$\begin{aligned}
 y &= 3x - 4 \\
 x &= \frac{y + 4}{3} \\
 f^{-1}(x) &= \frac{x + 4}{3} \\
 \frac{x + 4}{3} &= x^2 - 2 \\
 \times 3 \quad x + 4 &= 3x^2 - 6 \quad \times 3 \\
 0 &= 3x^2 - x - 10 \\
 0 &= (3x + 5)(x - 2) \\
 x &= 2 \text{ or } x = -\frac{5}{3}
 \end{aligned}$$