



- 1 A bag contains  $2x$  green counters and  $(x + 4)$  yellow counters. A counter is taken at random.

The probability that it is green is  $\frac{2}{5}$ .

Work out the value of  $x$ .

(3 marks)

$$\text{total counters} = 2x + (x + 4) = 3x + 4$$

$$\frac{2x}{(3x + 4)} = \frac{2}{5}$$

cross-multiply

$$10x = 2(3x + 4)$$

$$10x = 6x + 8$$

$$\begin{array}{rcl} -6x & 4x & = 8 \end{array}$$

$$\begin{array}{rcl} \div 4 & x & = 2 \end{array}$$

$$\boxed{x = 2}$$

- 2 A bag contains  $n$  counters. 3 of the counters are red.

A counter is taken at random, replaced, and then a second counter is taken at random.

Write down an expression, in terms of  $n$ , for the probability that both counters are red.

(2 marks)

Each pick has  $P(\text{red}) = \frac{3}{n}$  because the counter is replaced

$$P(\text{both red}) = \frac{3}{n} \times \frac{3}{n} = \frac{9}{n^2}$$

$$\boxed{\frac{9}{n^2}}$$

3

A box contains  $n$  counters. 5 of the counters are green.

A counter is taken at random and **not** replaced. A second counter is then taken at random.

Write down an expression, in terms of  $n$ , for the probability that both counters are green.

(2 marks)

$$P(\text{first green}) = \frac{5}{n}$$

After one green counter is removed, 4 green counters remain out of  $n - 1$

$$P(\text{both green}) = \frac{5}{n} \times \frac{4}{n-1} = \frac{20}{n(n-1)}$$

$$\boxed{\frac{20}{n(n-1)}}$$

4

A biased coin is flipped twice. The flips are independent.

The probability of getting heads on both flips is 0.64.

Work out the probability of getting heads on one flip of the coin.

(2 marks)

$$\text{Let } P(\text{heads}) = p. \text{ Then } p \times p = 0.64$$

$$p = \sqrt{0.64} = 0.8 \text{ (taking the positive root, since a probability cannot be negative)}$$

$$\boxed{0.8}$$

5

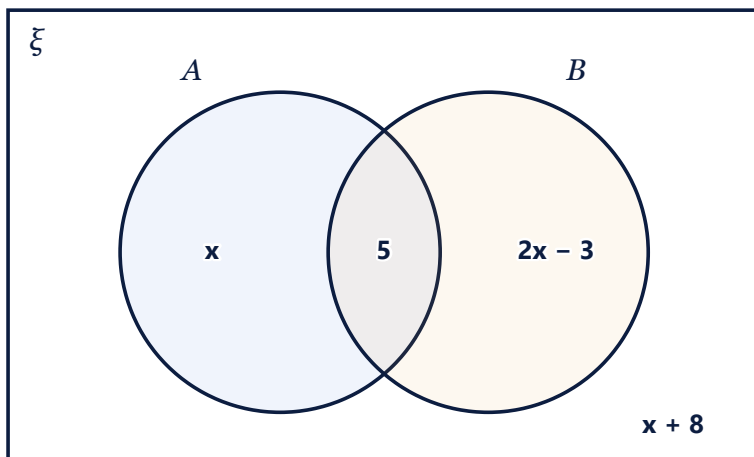
50 people were asked whether they read fiction (A) or non-fiction (B).

The Venn diagram regions contain these numbers of people:

A only:  $x$     $A \cap B$ : 5   B only:  $2x - 3$    neither:  $x + 8$

a. Work out the value of  $x$ .

b. One of the 50 people is chosen at random. Work out the probability that this person reads non-fiction.



(4 marks)

The four regions sum to 50

$$x + 5 + (2x - 3) + (x + 8) = 50$$

$$4x + 10 = 50, \text{ so } x = 10$$

The regions are 10, 5, 17 and 18 (check

$$10 + 5 + 17 + 18 = 50)$$

Number who read non-fiction =  $5 + 17 = 22$ , so the probability =

$$\frac{22}{50} = \frac{11}{25}$$

6

Two independent events A and B are such that

$$P(B) = 2 \times P(A)$$

$$P(A \text{ and } B) = \frac{1}{8}$$

Find  $P(A)$ .

(4 marks)

$$\text{Let } P(A) = x, \text{ so } P(B) = 2x$$

A and B are independent, so  $P(A \text{ and } B) = x \times 2x = 2x^2$

$$2x^2 = \frac{1}{8}, \text{ so } x^2 = \frac{1}{16}$$

$$x = \frac{1}{4} \text{ (taking the positive root), so } P(A) = \frac{1}{4}$$

$$\boxed{\frac{1}{4}}$$

7

A jar contains  $n$  sweets. 6 of the sweets are orange. The rest are lemon.

Hana takes two sweets at random, without replacement.

The probability that both sweets are orange is  $\frac{1}{3}$ .

a. Show that  $n^2 - n - 90 = 0$ .

b. Find the value of  $n$ .

(5 marks)

$$P(\text{both orange}) = \frac{6}{n} \times \frac{5}{n-1} = \frac{30}{n(n-1)}$$

Setting this equal to  $\frac{1}{3}$  gives  $\frac{30}{n(n-1)} = \frac{1}{3}$ , so  $n(n-1) = 90$

Expanding

$$n^2 - n - 90 = 0, \text{ as required}$$

Factorising

$$(n-10)(n+9) = 0, \text{ so } n = 10 \text{ or } n = -9$$

$n$  must be a positive number of sweets, so  $n = 10$

Check

$$\frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$$

8

A bag contains  $x$  green counters and 8 white counters.

Two counters are taken at random, without replacement.

The probability that both counters are white is  $\frac{14}{33}$ .

Work out the value of  $x$ .

(5 marks)

The bag holds  $x + 8$  counters, so  $P(\text{both white}) = \frac{8}{x+8} \times \frac{7}{x+7} = \frac{56}{(x+8)(x+7)}$

Setting this equal to  $\frac{14}{33}$

$$\frac{56}{(x+8)(x+7)} = \frac{14}{33}$$

$$(x+8)(x+7) = 56 \times \frac{33}{14} = 132$$

$$x^2 + 15x + 56 = 132, \text{ so } x^2 + 15x - 76 = 0$$

Factorising

$$(x+19)(x-4) = 0, \text{ so } x = 4 \text{ (} x \text{ must be positive)}$$

Check

$$\frac{8}{12} \times \frac{7}{11} = \frac{56}{132} = \frac{14}{33}$$

4

9

A bag contains only red counters and green counters.

The probability of taking a red counter at random is  $\frac{2}{5}$ .

10 red counters are then removed from the bag.

The probability of taking a red counter at random is now  $\frac{1}{4}$ .

Work out the number of counters that were in the bag originally.

(5 marks)

Let the original total be  $t$ . The number of red counters is  $\frac{2t}{5}$

After removing 10 red counters there are  $\frac{2t}{5} - 10$  red counters and  $t - 10$  counters in total

$$\frac{\frac{2t}{5} - 10}{t - 10} = \frac{1}{4}, \text{ so } 4(\frac{2t}{5} - 10) = t - 10$$

$$\frac{8t}{5} - 40 = t - 10, \text{ so } \frac{3t}{5} = 30 \text{ and } t = 50$$

Check

originally 20 red out of 50 ( $P = \frac{2}{5}$ ); after removal 10 red

out of 40 ( $P = \frac{1}{4}$ )

50

10

A bag contains 4 red counters and  $x$  blue counters. Two counters are taken at random without replacement.

a. Show that the probability of taking one red counter and one blue counter is  $\frac{8x}{(x+4)(x+3)}$ .

b. The probability of taking one counter of each colour is  $\frac{8}{15}$ . Show that  $x^2 - 8x + 12 = 0$ .

c. Solve  $x^2 - 8x + 12 = 0$  to find the possible values of  $x$ .

(6 marks)

a) two orders: red then blue, blue then red

$$P(R,B) = \frac{4}{(x+4)} \times \frac{x}{(x+3)}$$

$$P(B,R) = \frac{x}{(x+4)} \times \frac{4}{(x+3)}$$

$$\begin{aligned} P(\text{one of each}) &= 2 \times \frac{4x}{((x+4)(x+3))} \\ &= \frac{8x}{((x+4)(x+3))} \end{aligned}$$

b)

$$\begin{aligned} \frac{8x}{((x+4)(x+3))} &= \frac{8}{15} \\ 15x &= (x+4)(x+3) \\ 15x &= x^2 + 7x + 12 \\ -15x \quad \quad \quad -15x \\ 0 &= x^2 - 8x + 12 \end{aligned}$$

c) factorise

$$(x-2)(x-6) = 0$$

$$x = 2 \text{ or } x = 6$$

11

A bag contains  $x$  black counters and 6 white counters.

Two counters are taken at random, without replacement.

The probability that the two counters are different colours is  $\frac{1}{2}$ .

Find the possible values of  $x$ .

(6 marks)

The bag holds  $x + 6$  counters

$$P(\text{black then white}) = \frac{x}{x+6} \times \frac{6}{x+5} \text{ and } P(\text{white then black}) = \frac{6}{x+6} \times \frac{x}{x+5}$$

$$P(\text{different colours}) = \frac{12x}{(x+6)(x+5)} = \frac{1}{2}$$

$$24x = (x+6)(x+5) = x^2 + 11x + 30$$

$$x^2 - 13x + 30 = 0, \text{ so } (x-3)(x-10) = 0$$

$$x = 3 \text{ or } x = 10$$

$$\text{Check } x = 3: 2 \times 3 \times \frac{6}{9 \times 8} = \frac{36}{72} = \frac{1}{2}. \text{ Check } x = 10: 2 \times 10 \times \frac{6}{16 \times 15} = \frac{120}{240} = \frac{1}{2}$$

$$x = 3 \text{ or } x = 10$$

12

There are  $n$  counters in a box. 8 of the counters are orange. The rest are brown.

Meg takes two counters at random, without replacement.

The probability that both counters are orange is  $\frac{4}{15}$ .

a. Show that  $n^2 - n - 210 = 0$ .

b. Find the value of  $n$ .

c. Work out the probability that both counters Meg takes are brown.

(8 marks)

$$P(\text{both orange}) = \frac{8}{n} \times \frac{7}{n-1} = \frac{56}{n(n-1)}$$

Setting this equal to  $\frac{4}{15}$

$$\frac{56}{n(n-1)} = \frac{4}{15}, \text{ so } n(n-1) = 56 \times \frac{15}{4} = 210$$

Expanding

$$n^2 - n - 210 = 0, \text{ as required}$$

Factorising

$$(n-15)(n+14) = 0, \text{ so } n = 15 \text{ (} n \text{ must be positive)}$$

Check

$$\frac{8}{15} \times \frac{7}{14} = \frac{56}{210} = \frac{4}{15}$$

There are  $15 - 8 = 7$  brown counters

$$P(\text{both brown}) = \frac{7}{15} \times \frac{6}{14} = \frac{42}{210} = \frac{1}{5}$$