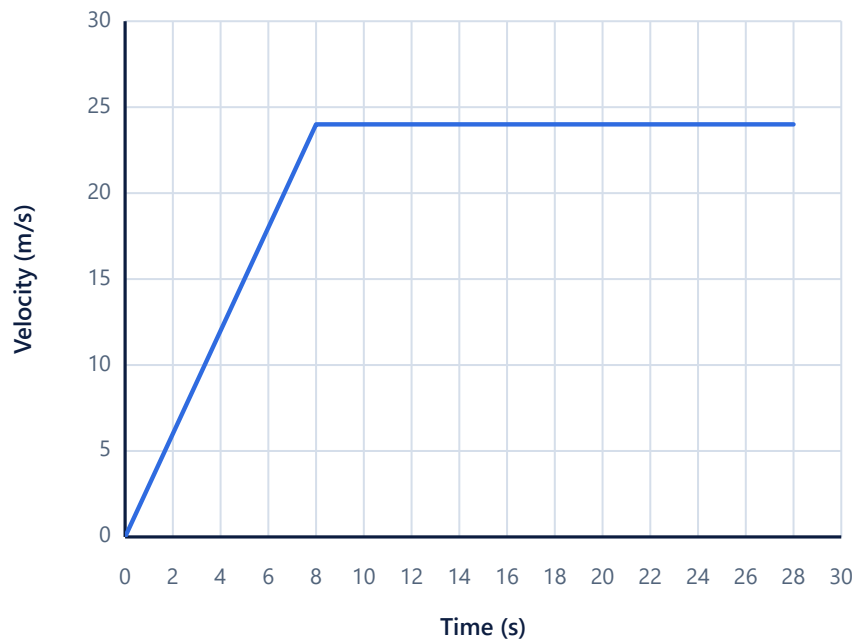




- 1** A car starts from rest and accelerates uniformly to a velocity of 24 m/s in 8 seconds. It then travels at a constant velocity of 24 m/s for a further 20 seconds.
- Work out the acceleration of the car during the first 8 seconds.
 - Work out the total distance travelled by the car in the 28 seconds.



(3 marks)

a) Acceleration = change in velocity divided by time = $\frac{24}{8} = 3$
m/s²

b) Distance = area under the velocity-time graph

First 8 seconds (triangle)

$$\frac{1}{2} \times 8 \times 24 = 96 \text{ m}$$

Next 20 seconds (rectangle)

$$20 \times 24 = 480 \text{ m}$$

$$\text{Total distance} = 96 + 480 = 576 \text{ m}$$

2

Water flows into an empty tank. The rate of flow increases uniformly from 0 litres per minute to 12 litres per minute during the first 5 minutes. It then stays at 12 litres per minute for a further 10 minutes.

- Work out the total volume of water in the tank after 15 minutes.
- The tank holds 200 litres. The flow stays at 12 litres per minute. Work out how many more minutes it takes for the tank to fill. Give your answer in minutes and seconds.

(4 marks)

a) Area under the graph = volume

$$\text{Triangle: } \frac{1}{2} \times 5 \times 12 = 30$$

$$\text{Rectangle: } 10 \times 12 = 120$$

$$30 + 120 = 150 \text{ Litres}$$

b)

$$200 - 150 = 50 \text{ litres still needed}$$

$$\text{time} = \frac{50}{12} = 4 \frac{1}{6} \text{ minutes}$$

$$\frac{1}{6} \text{ of a minute} = 10 \text{ seconds}$$

$$4 \text{ minutes } 10 \text{ seconds}$$

3

The running cost of a machine is measured in pounds per hour. For the first 2 hours the cost is constant at £6 per hour. Over the next 4 hours the cost increases uniformly from £6 per hour to £14 per hour.

- Work out the total cost of running the machine for the 6 hours.
- Work out the mean cost per hour over the 6 hours. Give your answer to the nearest penny.

(4 marks)

a) Area under the graph = total cost

$$\text{Rectangle: } 2 \times 6 = 12$$

$$\text{Trapezium: } \frac{1}{2} \times (6 + 14) \times 4 = 40$$

$$12 + 40 = £52$$

b) Mean = total $\div$ time

$$52 \div 6 = 8.666...$$

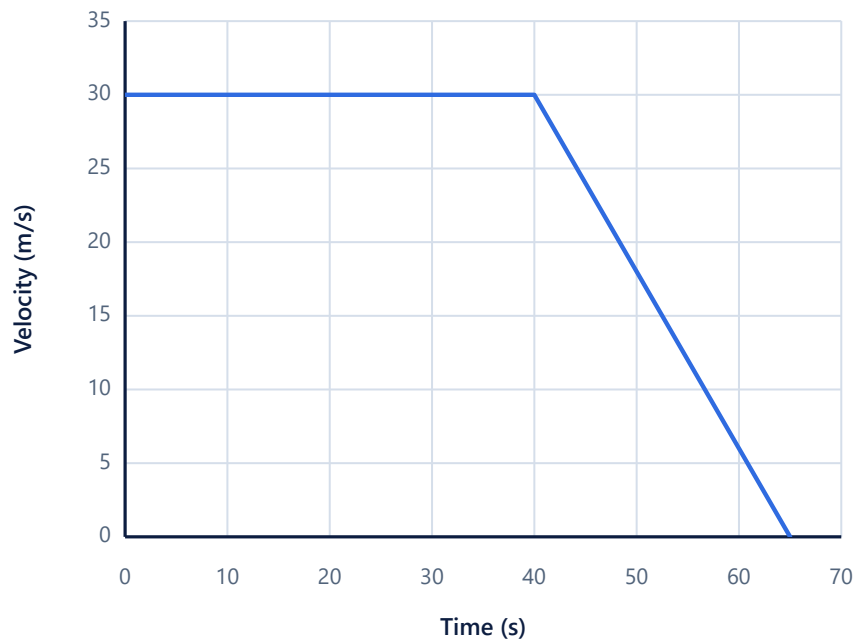
$$£8.67 \text{ per hour}$$

4

A train travels at a constant velocity of 30 m/s for 40 seconds.

It then decelerates uniformly, coming to rest in a further 25 seconds.

- Work out the deceleration of the train.
- Work out the total distance the train travels in the 65 seconds.



(4 marks)

$$a) \text{ Deceleration} = \frac{30}{25} = 1.2 \text{ m/s}^2$$

$$b) \text{ Distance} = \text{area under the velocity-time graph}$$

Constant phase (rectangle)

$$40 \times 30 = 1200 \text{ m}$$

Decelerating phase (triangle)

$$\frac{1}{2} \times 25 \times 30 = 375 \text{ m}$$

$$\text{Total} = 1200 + 375 = 1575 \text{ m}$$

5

A motorbike accelerates from rest. Its speed-time graph is a smooth curve that gets less steep throughout the first 12 seconds. The table gives the speed v m/s at time t seconds.

t : 0, 4, 8, 12 v : 0, 7, 11, 13

a. Use 3 strips of equal width to estimate the distance travelled in the first 12 seconds.

b. Amir says, "The acceleration at $t = 8$ is $11 \div 8 = 1.375 \text{ m/s}^2$." Explain why Amir is wrong.

(4 marks)

a) Trapezium strips of width 4: $\frac{1}{2} \times 4 \times (0 + 7) + \frac{1}{2} \times 4 \times (7 + 11) + \frac{1}{2} \times 4 \times (11 + 13)$

$$= 14 + 36 + 48 = 98 \text{ m}$$

b) 11 divided by 8 is the gradient of the straight line from the origin to (8, 11): the average acceleration over the first 8 seconds

The acceleration at the instant $t = 8$ is the gradient of the tangent to the curve at $t = 8$. Because the curve is getting less steep, this tangent gradient is smaller than 1.375 m/s^2 , so Amir's method is wrong

6

A train travels between two stations. It accelerates uniformly from rest to 18 m/s in 6 seconds, travels at a constant 18 m/s for 14 seconds, then decelerates uniformly to rest in 10 seconds.

a. Work out the total distance between the two stations.

b. Work out the average speed for the whole journey.

c. Write the average speed as a percentage of the maximum speed. Give your answer correct to 1 decimal place.

(5 marks)

a) Distance = area under the graph

$$\text{Acceleration: } \frac{1}{2} \times 6 \times 18 = 54$$

$$\text{Constant speed: } 14 \times 18 = 252$$

$$\text{Deceleration: } \frac{1}{2} \times 10 \times 18 = 90$$

$$54 + 252 + 90 = 396 \text{ m}$$

$$\text{b) Total time} = 6 + 14 + 10 = 30 \text{ s}$$

$$\text{average speed} = \frac{396}{30}$$

$$13.2 \text{ m/s}$$

c)

$$= \frac{13.2}{18} \times 100$$

$$73.3\%$$

7

A lorry starts from rest and accelerates uniformly for 10 seconds until it reaches a velocity of 20 m/s. It then travels at a constant velocity of 20 m/s for T seconds. The lorry travels a total distance of 700 m. Work out the total time of the journey.

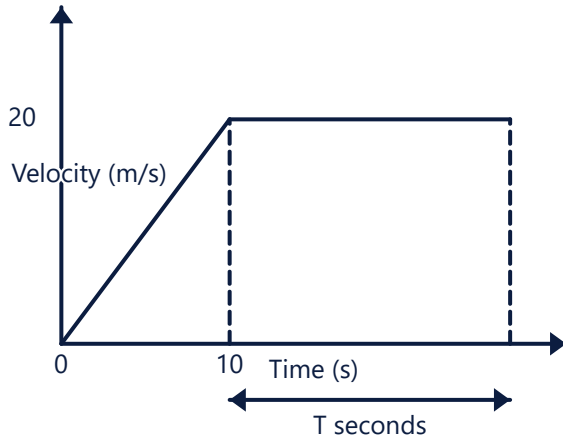


Diagram NOT accurately drawn

(5 marks)

Distance in the first 10 seconds (triangle)

$$\frac{1}{2} \times 10 \times 20 = 100 \text{ m}$$

Distance remaining at constant velocity

$$700 - 100 = 600 \text{ m}$$

Time at 20 m/s

$$\frac{600}{20} = 30 \text{ seconds, so } T = 30$$

$$\text{Total time} = 10 + 30 = 40 \text{ seconds}$$

40 seconds

8

The table shows the distance, s metres, travelled by a cyclist from a start line, t seconds after she starts to accelerate.

t	0	2	4	6	8	10
s	0	6	20	42	72	110

- Work out the average speed of the cyclist over the first 10 seconds.
- Estimate the cyclist's speed at $t = 6$ by working out the gradient of the chord joining the points at $t = 4$ and $t = 8$.
- What does the gradient of the distance-time graph represent?

(4 marks)

a) Average speed = distance $\div$ time

$$= \frac{110}{10}$$

$$11 \text{ m/s}$$

b) Gradient of the chord from $t = 4$ to $t = 8$

$$\frac{72 - 20}{8 - 4} = \frac{52}{4}$$

$$13 \text{ m/s}$$

c) Gradient of a distance-time graph

speed (rate of change of distance)

9

A car makes a journey in three stages.

Stage 1: it accelerates uniformly from rest to a velocity of V m/s in 12 seconds.

Stage 2: it travels at the constant velocity V m/s for 48 seconds.

Stage 3: it decelerates uniformly from V m/s to rest in 20 seconds.

The total distance of the journey is 960 m.

- Work out the value of V .
- Work out the acceleration of the car during stage 1.

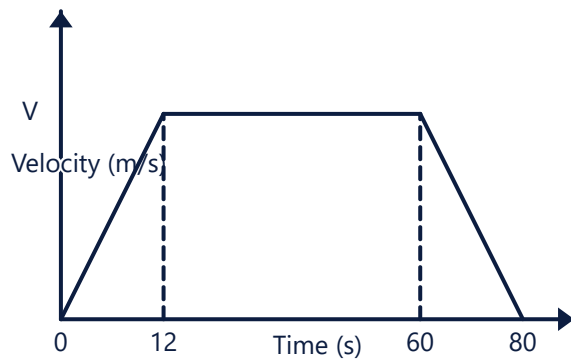


Diagram NOT accurately drawn

(6 marks)

a) Total distance = area under the velocity-time graph (a trapezium made of a triangle, a rectangle and a triangle)

Stage 1

$$\frac{1}{2} \times 12 \times V = 6V. \text{ Stage 2: } 48V. \text{ Stage 3: } \frac{1}{2} \times 20 \times V =$$

$$10V$$

$$6V + 48V + 10V = 64V = 960, \text{ so } V = 15$$

Check

$$6(15) + 48(15) + 10(15) = 90 + 720 + 150 = 960$$

$$\text{b) Acceleration in stage 1} = \frac{15}{12} = 1.25 \text{ m/s}^2$$

10

The table shows the speed, v m/s, of a motorbike t seconds after it starts to accelerate. The graph of v against t is a smooth curve that gets less steep.

t	0	5	10	15	20
v	0	8	13	16	17

- Use four trapezia to estimate the distance travelled in the first 20 seconds.
- Is your answer an underestimate or an overestimate? Give a reason.

(4 marks)

a) Four trapezia of width 5

$$\frac{1}{2} \times 5 \times (0 + 8) = 20$$

$$\frac{1}{2} \times 5 \times (8 + 13) = 52.5$$

$$\frac{1}{2} \times 5 \times (13 + 16) = 72.5$$

$$\frac{1}{2} \times 5 \times (16 + 17) = 82.5$$

$$20 + 52.5 + 72.5 + 82.5 = 227.5 \text{ m}$$

b) The curve is above each straight chord

Underestimate