

Turning points by completing the square

Worked solutions

GCSE Maths topic practice



This topic online

1 $y = x^2 + 6x + 4$

By completing the square, find the coordinates of the turning point and say whether it is a maximum or a minimum.

(3 marks)

$$\begin{aligned} x^2 + 6x + 4 &= (x + 3)^2 - 9 + 4 \\ &= (x + 3)^2 - 5 \end{aligned}$$

Turning point: $x + 3 = 0$

$(-3, -5)$, a minimum

2 $y = 3x^2 - 12x + 5$

Write y in the form $a(x + b)^2 + c$ and write down the coordinates of the turning point.

(3 marks)

$$\begin{aligned} y &= 3(x^2 - 4x) + 5 \\ &= 3[(x - 2)^2 - 4] + 5 \\ &= 3(x - 2)^2 - 12 + 5 \\ &= 3(x - 2)^2 - 7 \end{aligned}$$

Turning point $(2, -7)$

3 The curve **C** has equation $y = (x - 3)^2 + 5$

Write down the coordinates of the turning point of **C**.

(1 mark)

A squared real number cannot be negative. $(x - 3)^2$ is smallest when $x - 3 = 0$, so $x = 3$

At $x = 3$, $y = 0 + 5 = 5$. The curve therefore has its minimum (turning point) at $(3, 5)$

$(3, 5)$

4

a. Write $x^2 - 6x + 11$ in the form $(x - a)^2 + b$, where a and b are integers.b. Hence find the coordinates of the turning point of the curve $y = x^2 - 6x + 11$

(3 marks)

$$a) (x - 3)^2 = x^2 - 6x + 9, \text{ so } x^2 - 6x + 11 = (x - 3)^2 + 2$$

a) Check by expanding

$$x^2 - 6x + 9 + 2 = x^2 - 6x + 11$$

b) The minimum of $(x - 3)^2 + 2$ is when $x = 3$, giving the turning point $(3, 2)$

5

By completing the square, find the coordinates of the minimum point of the curve

$$y = x^2 + 10x - 2$$

(3 marks)

Complete the square

$$x^2 + 10x - 2 = (x + 5)^2 - 25 - 2 = (x + 5)^2 - 27$$

Check by expanding

$$x^2 + 10x + 25 - 27 = x^2 + 10x - 2$$

The minimum is when $x = -5$, so the minimum point is $(-5, -27)$

$$\boxed{(-5, -27)}$$

6

The curve $y = x^2 + bx + c$ passes through the points $(0, 7)$ and $(1, 4)$.

Find the coordinates of the turning point of the curve.

(4 marks)

At $(0, 7)$

$$c = 7$$

At $(1, 4)$

$$1 + b + 7 = 4, \text{ so } b = -4$$

The curve is $y = x^2 - 4x + 7$

Complete the square

$$x^2 - 4x + 7 = (x - 2)^2 + 3$$

Check by expanding

$$x^2 - 4x + 4 + 3 = x^2 - 4x + 7$$

The turning point is $(2, 3)$

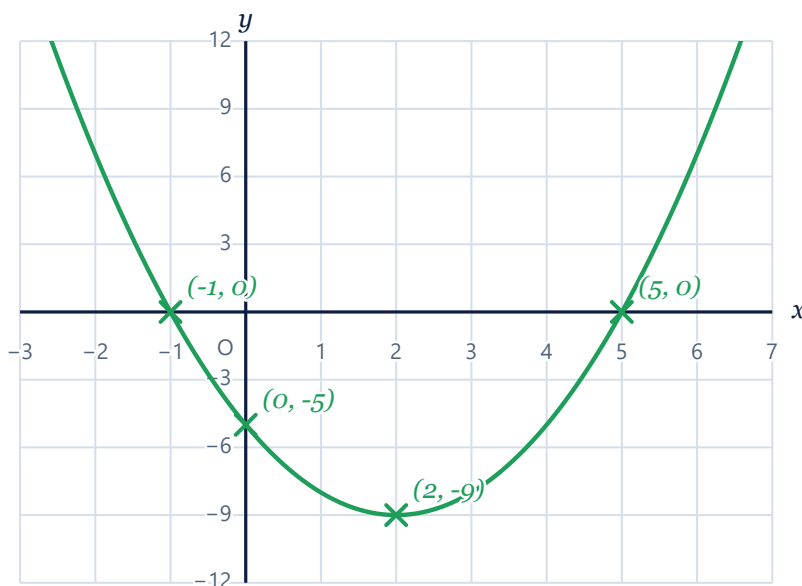
$$\boxed{(2, 3)}$$

7

The curve **C** has equation $y = x^2 - 4x - 5$

a. Write $x^2 - 4x - 5$ in the form $(x - a)^2 + b$

b. Sketch **C**, showing clearly the coordinates of the turning point and the coordinates of the points where **C** crosses the axes.



(5 marks)

$$a) x^2 - 4x - 5 = (x - 2)^2 - 4 - 5 = (x - 2)^2 - 9$$

a) Check by expanding

$$x^2 - 4x + 4 - 9 = x^2 - 4x - 5$$

b) The turning point is $(2, -9)$

b) For the x-intercepts, factorise

$$x^2 - 4x - 5 = (x - 5)(x + 1), \text{ so the curve crosses at } (5, 0) \text{ and } (-1, 0)$$

b) When $x = 0$, $y = -5$, so the y-intercept is $(0, -5)$

b) Draw a U-shaped parabola through these points with its minimum at $(2, -9)$

8

a. Show that $x^2 + 5x + 9$ can be written as $(x + \frac{5}{2})^2 + \frac{11}{4}$

b. Write down the minimum value of $x^2 + 5x + 9$

c. Hence explain why the equation $x^2 + 5x + 9 = 0$ has no real solutions.

(5 marks)

a) Halve the coefficient of x

$$(x + \frac{5}{2})^2 = x^2 + 5x + \frac{25}{4}$$

a) So $x^2 + 5x + 9 = (x + \frac{5}{2})^2 - \frac{25}{4} + 9 = (x + \frac{5}{2})^2 + \frac{11}{4}$, since 9
 $= \frac{36}{4}$

b) The square term is at least 0, so the minimum value is $\frac{11}{4}$,
 when $x = -\frac{5}{2}$

c) The expression is always at least $\frac{11}{4}$, which is greater than 0, so it can never equal 0 and the equation has no real solutions

9

$$f(x) = x^2 - 4x + 7$$

a. Write $f(x)$ in the form $(x - a)^2 + b$ and hence write down the coordinates of the turning point of $y = f(x)$

b. Write down the coordinates of the turning point of $y = f(x - 3)$

c. Find the equation of $y = f(-x)$, giving your answer in the form $y = x^2 + bx + c$

(4 marks)

a) Complete the square

$$x^2 - 4x + 7 = (x - 2)^2 - 4 + 7 = (x - 2)^2 + 3. \text{ The turning point is } (2, 3)$$

b) $y = f(x - 3)$ translates the graph 3 units to the right, so the turning point moves to (5, 3)

c) $f(-x) = (-x)^2 - 4(-x) + 7 = x^2 + 4x + 7$, so $y = x^2 + 4x + 7$

10 $y = -x^2 + 6x - 4$

By completing the square, find the coordinates of the turning point. State whether it is a maximum or a minimum.

(3 marks)

$$\begin{aligned}y &= -(x^2 - 6x) - 4 \\&= -[(x - 3)^2 - 9] - 4 \\&= -(x - 3)^2 + 9 - 4 \\&= -(x - 3)^2 + 5\end{aligned}$$

$(3, 5)$, a maximum

11 A ball is thrown upwards. Its height, h metres, t seconds after it is thrown is given by

$$h = 1.5 + 20t - 5t^2$$

By completing the square, work out the maximum height of the ball and the time at which it reaches this height.

(4 marks)

$$\begin{aligned}h &= -5t^2 + 20t + 1.5 \\&= -5(t^2 - 4t) + 1.5 \\&= -5[(t - 2)^2 - 4] + 1.5 \\&= -5(t - 2)^2 + 20 + 1.5 \\&= -5(t - 2)^2 + 21.5\end{aligned}$$

Max height 21.5 m at $t = 2$ s

12 The graph of $y = x^2 + bx + c$ has a turning point at $(3, -2)$.

Find the value of b and the value of c .

(4 marks)

Turning point $(3, -2)$

$$\begin{aligned}y &= (x - 3)^2 - 2 \\&= x^2 - 6x + 9 - 2 \\&= x^2 - 6x + 7\end{aligned}$$

$b = -6, c = 7$