

Finding the n th term of a linear sequence

Worked solutions

GCSE Maths topic practice



This topic online

- 1 Here are the first four terms of an arithmetic sequence.

5 9 13 17

Find an expression, in terms of n , for the n th term of the sequence.

(2 marks)

n is the position in the sequence, starting with $n = 1$. The n th-term rule lets you calculate a term directly from its position

The common difference is 4, so the n th term contains $4n$

The sequence $4n$ gives 4, 8, 12, 16, which is 1 less than each term

The n th term is $4n + 1$

Check

$$\text{when } n = 3, 4(3) + 1 = 13$$

$$4n + 1$$

- 2 Here are the first four terms of an arithmetic sequence.

20 17 14 11

Find an expression, in terms of n , for the n th term of the sequence.

(2 marks)

The common difference is -3, so the n th term contains $-3n$

The sequence $-3n$ gives -3, -6, -9, -12, which is 23 less than each term

The n th term is $23 - 3n$

Check

$$\text{when } n = 2, 23 - 6 = 17$$

$$23 - 3n$$

- 3 Here are the first four terms of an arithmetic sequence.

7 12 17 22

Explain why 81 is not a term of this sequence.

(2 marks)

The n th term is $5n + 2$

If $5n + 2 = 81$ then $5n = 79$, so $n = 15.8$, which is not a whole number

Alternatively, every term ends in a 2 or a 7, but 81 ends in a 1
So 81 is not a term of the sequence

- 4 The n th term of a sequence is $6n - 1$

Write down the first three terms of the sequence.

(2 marks)

$$\text{When } n = 1: 6(1) - 1 = 5$$

$$\text{When } n = 2: 6(2) - 1 = 11$$

$$\text{When } n = 3: 6(3) - 1 = 17$$

5, 11, 17

- 5 Here are the first four terms of an arithmetic sequence.

3 10 17 24

Find an expression, in terms of n , for the n th term of the sequence.

(2 marks)

The common difference is 7, so the n th term contains $7n$

The sequence $7n$ gives 7, 14, 21, 28, which is 4 more than each term

The n th term is $7n - 4$

Check

$$\text{when } n = 4, 7(4) - 4 = 24$$

$7n - 4$

6

Matchsticks are used to make a row of squares. One square uses 4 matchsticks, two squares joined in a row use 7 matchsticks, and three squares joined in a row use 10 matchsticks. The pattern continues.

- Find an expression, in terms of n , for the number of matchsticks needed for n squares.
- How many matchsticks are needed for 20 squares?

(3 marks)

a)

4, 7, 10: difference 3

$3n$: 3, 6, 9

add 1 to get 4, 7, 10

n th term = $3n + 1$

b)

$$3 \times 20 + 1 = 61$$

a) $3n + 1$ b) 61

7

Here are the first four terms of an arithmetic sequence.

4 7 10 13

- Write down the next term of the sequence.
- Find an expression, in terms of n , for the n th term of the sequence.

(3 marks)

a) The common difference is 3, so the next term is $13 + 3 = 16$

b) The n th term contains $3n$, and $3n$ gives 3, 6, 9, 12, which is 1 less than each term

The n th term is $3n + 1$

8

Here are the first four terms of a sequence.

7 11 15 19

- Find an expression, in terms of n , for the n th term.
- Which term in the sequence has a value of 99?

(3 marks)

a)

difference = 4

$4n$: 4, 8, 12, 16

+3 gives 7, 11, 15, 19

n th term = $4n + 3$

b)

$$\begin{array}{rcl} 4n + 3 & = & 99 \\ -3 & & \\ \hline 4n & = & 96 \\ \div 4 & & \div 4 \\ n & = & 24 \end{array}$$

a) $4n + 3$ b) 24th term

9

The n th term of a sequence is $5n - 3$

- Work out the 12th term of the sequence.
- Is 122 a term of the sequence? You must show how you get your answer.

(3 marks)

$$a) 5(12) - 3 = 60 - 3 = 57$$

$$b) \text{ Solve } 5n - 3 = 122: 5n = 125, \text{ so } n = 25$$

$n = 25$ is a whole number, so 122 is the 25th term of the sequence

10

Here are the first four terms of an arithmetic sequence.

8 15 22 29

- Find an expression, in terms of n , for the n th term of the sequence.
- Explain why 150 is not a term of the sequence.

(4 marks)

a) The common difference is 7, and $7n$ gives 7, 14, 21, 28, which is 1 less than each term

The n th term is $7n + 1$

$$b) \text{ If } 7n + 1 = 150 \text{ then } 7n = 149$$

149 divided by 7 is not a whole number, so 150 is not a term

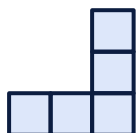
11

Here are the first three patterns in a sequence made from grey square tiles.

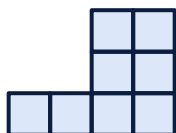
Pattern 1 uses 5 tiles. Pattern 2 uses 8 tiles. Pattern 3 uses 11 tiles.

The number of tiles goes up by the same amount from each pattern to the next.

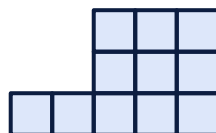
- Work out the number of tiles in pattern 5.
- Write down an expression, in terms of n , for the number of tiles in pattern n .
- Explain why there is no pattern in this sequence that uses exactly 100 tiles.



Pattern 1



Pattern 2



Pattern 3

(5 marks)

a) The pattern goes up by 3 tiles each time

pattern 4 uses $11 + 3 = 14$ tiles, pattern 5 uses $14 + 3 = 17$ tiles

b) The common difference is 3, so the expression starts $3n$. For $n = 1$, $3n = 3$, but pattern 1 has 5 tiles, so add 2: $3n + 2$

b) Check

$n = 2$ gives 8 and $n = 3$ gives 11, both correct

c) If $3n + 2 = 100$ then $3n = 98$. $98 \div 3$ is not a whole number, so no pattern uses exactly 100 tiles

12

The first term of an arithmetic sequence is 3.

The term-to-term rule is add 6.

- Work out the 5th term of the sequence.
- Find an expression, in terms of n , for the n th term of the sequence.
- Explain why every term of the sequence is an odd number.

(4 marks)

a) The terms are 3, 9, 15, 21, 27, so the 5th term is 27

b) The common difference is 6, and $6n$ gives 6, 12, 18, which is 3 more than each term, so the n th term is $6n - 3$

c) $6n$ is a multiple of 6, so it is always even

An even number minus 3 is always odd, so every term is odd

13

The n th term of a sequence is $50 - 6n$.

- Write down the first three terms of the sequence.
- Work out the first term of the sequence that is negative.

(4 marks)

a)

$$n = 1: 50 - 6 = 44$$

$$n = 2: 50 - 12 = 38$$

$$n = 3: 50 - 18 = 32$$

b) negative when $6n > 50$

$$n = 8: 50 - 48 = 2 \text{ (positive)}$$

$$n = 9: 50 - 54 = -4$$

a) 44, 38, 32 b) -4

14

Here are the first four terms of an arithmetic sequence.

100 93 86 79

- Write down the next two terms of the sequence.
- Find an expression, in terms of n , for the n th term of the sequence.
- Explain why 0 is not a term of the sequence.

(4 marks)

a) The common difference is -7 , so the next terms are $79 - 7 = 72$ and $72 - 7 = 65$

b) The n th term contains $-7n$, and $-7n$ gives $-7, -14, -21$, which is 107 less than each term, so the n th term is $107 - 7n$

Check

$$\text{when } n = 1, 107 - 7 = 100$$

$$\text{c) If } 107 - 7n = 0 \text{ then } 7n = 107$$

107 divided by 7 is not a whole number, so 0 is not a term