



1 Here are the side lengths of four triangles.

Triangle A: 5 cm, 7 cm, 9 cm

Triangle B: 5 cm, 7 cm, 8 cm

Triangle C: 9 cm, 7 cm, 5 cm

Triangle D: 5 cm, 6 cm, 9 cm

Write down the letters of the two triangles that are congruent to each other.

(1 mark)

Congruent triangles have the same shape and size. SSS (side-side-side) says that three matching side lengths are enough to establish this, even if one triangle is rotated or reflected

Triangle A has sides 5 cm, 7 cm and 9 cm. Triangle C has sides 9 cm, 7 cm and 5 cm, which is the same set of three lengths

Triangle B (5, 7, 8) and Triangle D (5, 6, 9) each differ from this set, so the congruent pair is A and C

A and C

2 In triangles PQR and XYZ,

$PQ = XY = 6$ cm

angle PQR = angle XYZ = 40°

$QR = YZ = 8$ cm

The two triangles are congruent.

Write down the congruence condition that shows the triangles are congruent.

(1 mark)

Two pairs of sides are equal

$PQ = XY$ and $QR = YZ$

The equal angles, angle PQR and angle XYZ, are between these pairs of sides at Q and at Y, so the angle is the included angle in each triangle

Two sides and the included angle match, so the condition is SAS

SAS

3

For each pair of triangles, state whether they must be congruent. If they must be, give the condition (SSS, SAS, ASA or RHS).

- Triangle ABC and triangle DEF : $AB = DE = 6$ cm, $BC = EF = 8$ cm, angle $ABC = \text{angle } DEF = 55^\circ$.
- Triangle GHI and triangle KLM : angle $G = \text{angle } K = 40^\circ$, angle $H = \text{angle } L = 75^\circ$, $GH = KL = 10$ cm.
- Triangle NPQ and triangle RST : the angles in each triangle are 50° , 60° and 70° .
- Triangle UVW and triangle XYZ : angle $V = \text{angle } Y = 90^\circ$, $UV = XZ = 13$ cm, $VW = YZ = 5$ cm.

(4 marks)

a)

$AB = DE$, $BC = EF$, angle $B = \text{angle } E$ (between them)

SAS

b)

angle $G = \text{angle } K$, $GH = KL$, angle $H = \text{angle } L$

ASA

c)

Only angles are equal - no side is given

Not necessarily congruent

d)

Right angle, hypotenuse $13 = 13$, side $5 = 5$

RHS

4

ABCD is a kite in which $AB = AD$ and $CB = CD$.

The diagonal AC is drawn.

Prove that triangle ABC is congruent to triangle ADC.

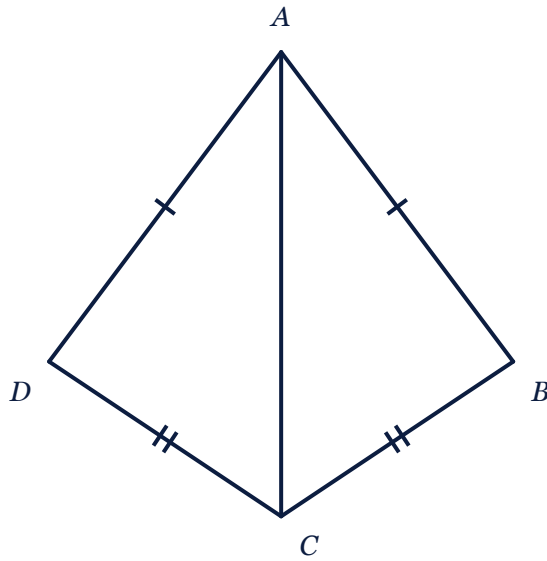


Diagram NOT accurately drawn

(3 marks)

$AB = AD$ because ABCD is a kite (given)

$CB = CD$ because ABCD is a kite (given)

AC is common to both triangles, so $AC = AC$

Three pairs of sides are equal, so triangle ABC is congruent to triangle ADC (SSS)

5

In triangle ABC, $AB = AC$.

The point D lies on BC so that AD bisects angle BAC.

Prove that triangle ABD is congruent to triangle ACD.

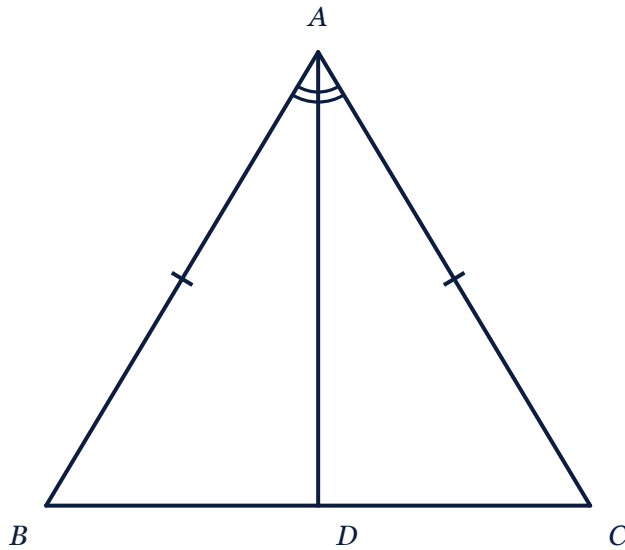


Diagram NOT accurately drawn

(3 marks)

$$AB = AC \text{ (given)}$$

$$\text{Angle } BAD = \text{angle } CAD \text{ because AD bisects angle BAC (given)}$$

$$AD \text{ is common to both triangles, so } AD = AD$$

In each triangle the equal angle is the included angle between the two equal sides (AB and AD in triangle ABD, AC and AD in triangle ACD)

So triangle ABD is congruent to triangle ACD (SAS)

6

ABC is an isosceles triangle with $AB = AC$.

M is the midpoint of BC .

- Prove that triangle ABM is congruent to triangle ACM .
- Hence prove that angle $ABC = \text{angle } ACB$.

(4 marks)

a)

$AB = AC$ (given)

$BM = CM$ (M is the midpoint of BC)

$AM = AM$ (common side)

SSS, so ABM is congruent to ACM

b)

Corresponding angles in congruent triangles are equal

angle $ABM = \text{angle } ACM$

angle $ABC = \text{angle } ACB$

7

In quadrilateral $ABCD$ the diagonal BD is drawn.

Angle $ABD = \text{angle } CDB$ and angle $ADB = \text{angle } CBD$.

Prove that triangle ABD is congruent to triangle CDB .

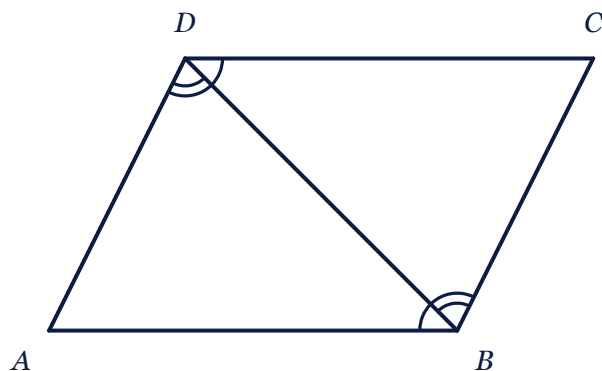


Diagram NOT accurately drawn

(3 marks)

Angle $ABD = \text{angle } CDB$ (given)

Angle $ADB = \text{angle } CBD$ (given)

BD is common to both triangles, so $BD = DB$

In triangle ABD the side BD lies between the angles ABD and ADB ; in triangle CDB the side DB lies between the corresponding angles CDB and CBD

Two pairs of angles and the included side are equal, so triangle ABD is congruent to triangle CDB (ASA)

8

OA and OB are two straight lines that meet at O.

The point P lies between the two lines.

M is the point on OA such that PM is perpendicular to OA.

N is the point on OB such that PN is perpendicular to OB.

$PM = PN$

Prove that triangle OMP is congruent to triangle ONP.

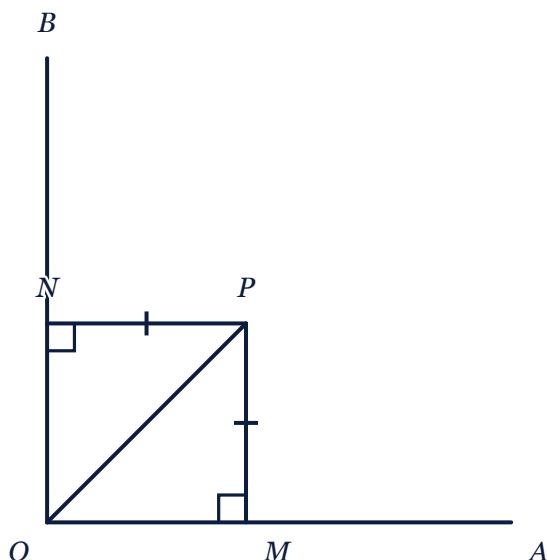


Diagram NOT accurately drawn

(3 marks)

Angle $OMP = \text{angle } ONP = 90^\circ$ because PM is perpendicular to OA and PN is perpendicular to OB (given)

OP is the hypotenuse of both right-angled triangles and is common, so $OP = OP$

$PM = PN$ (given)

A right angle, the hypotenuse and one other side are equal in each triangle, so triangle OMP is congruent to triangle ONP (RHS)

Note

it follows that angle $MOP = \text{angle } NOP$, so OP bisects the angle between the two lines

9

A and B are points on a circle with centre O.

AB is not a diameter. M is the midpoint of the chord AB.

Prove that OM bisects angle AOB.

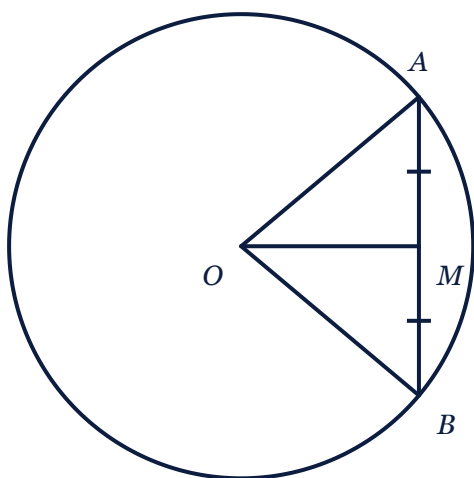


Diagram NOT accurately drawn

(3 marks)

$OA = OB$ because both are radii of the circle

$AM = BM$ because M is the midpoint of AB (given)

OM is common to both triangles, so $OM = OM$

So triangle OAM is congruent to triangle OBM (SSS)

Corresponding angles of congruent triangles are equal, so angle AOM = angle BOM

Therefore OM bisects angle AOB

10

M is the midpoint of the line XY. The line PM is perpendicular to XY.

Prove that $PX = PY$.

(3 marks)

$XM = YM$ (M is the midpoint)

angle PMX = angle PMY = 90° (PM perpendicular to XY)

$PM = PM$ (common)

SAS, so triangle PMX is congruent to triangle PMY

PX and PY are corresponding sides

$PX = PY$

11

Neither chord is a diameter.

AB and CD are chords of a circle with centre O.

$AB = CD$

- Prove that triangle OAB is congruent to triangle OCD.
- Hence explain why $\angle AOB = \angle COD$.

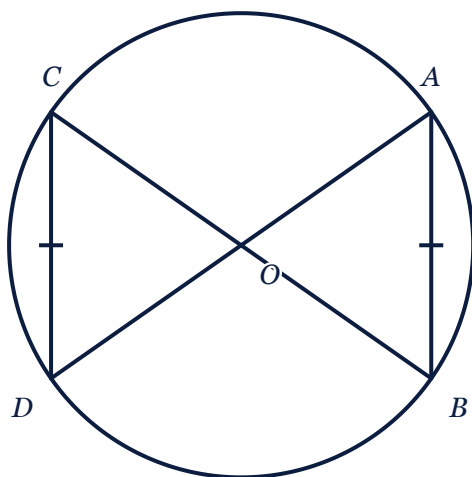


Diagram NOT accurately drawn

(4 marks)

- a) $OA = OC$ because both are radii of the circle
 $OB = OD$ because both are radii of the circle
 $AB = CD$ (given)

Three pairs of sides are equal, so triangle OAB is congruent to triangle OCD (SSS)

- b) Angle AOB in triangle OAB corresponds to angle COD in triangle OCD

Corresponding angles of congruent triangles are equal, so $\angle AOB = \angle COD$

This shows that equal chords subtend equal angles at the centre of a circle

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ABCD is a quadrilateral in which AB is parallel to DC and $AB = DC$.

The diagonal BD is drawn.

- Prove that triangle ABD is congruent to triangle CDB.
- Hence prove that $AD = CB$.

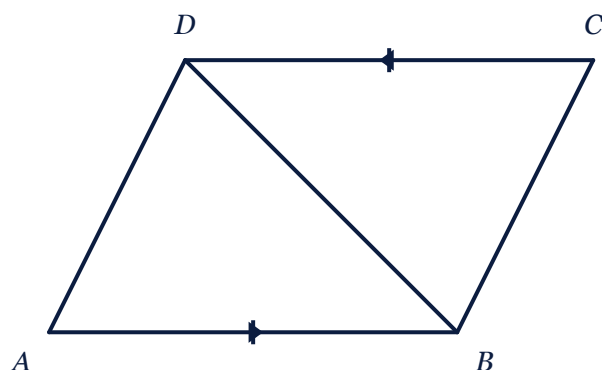


Diagram NOT accurately drawn

(4 marks)

a) $AB = DC$ (given)

Angle $ABD = \text{angle } BDC$ because they are alternate angles between the parallel lines AB and DC with transversal BD

BD is common to both triangles, so $BD = DB$

In triangle ABD the angle ABD is included between the sides AB and BD; in triangle CDB the angle CBD is included between the corresponding sides CD and DB

So triangle ABD is congruent to triangle CDB (SAS)

b) In the congruence, A corresponds to C, B corresponds to D and D corresponds to B, so the side AD corresponds to the side CB

Corresponding sides of congruent triangles are equal, so $AD = CB$

This proves that a quadrilateral with one pair of opposite sides equal and parallel is a parallelogram

13

ABCD is a parallelogram.

The diagonals AC and BD intersect at the point M.

Prove that the diagonals of the parallelogram bisect each other.

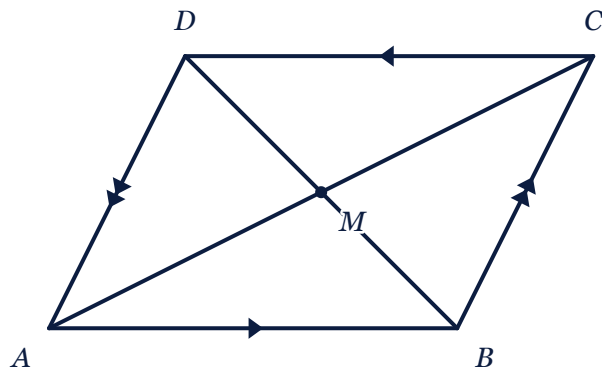


Diagram NOT accurately drawn

(5 marks)

AB is parallel to DC because ABCD is a parallelogram

Angle BAM = angle DCM because they are alternate angles between the parallel lines AB and DC with transversal AC

Angle ABM = angle CDM because they are alternate angles between the parallel lines AB and DC with transversal BD

AB = DC because opposite sides of a parallelogram are equal

In triangle ABM the side AB is included between the angles BAM and ABM; in triangle CDM the side DC is included between the corresponding angles DCM and CDM

So triangle ABM is congruent to triangle CDM (ASA)

In the congruence, A corresponds to C and B corresponds to D, so

AM = CM and BM = DM

Therefore M is the midpoint of both AC and BD, so the diagonals bisect each other