



1 Write each recurring decimal as a fraction in its simplest form.

a.  $0.\dot{3}$

b.  $0.\dot{8}$

(2 marks)

a.)

$$\begin{array}{rcl} x & = & 0.333... \\ 10x & = & 3.333... \\ 10x - x & & 9x = 3 \\ \div 9 & & x = \frac{3}{9} = \frac{1}{3} \end{array}$$

$$\boxed{\frac{1}{3}}$$

b.)

$$\begin{array}{rcl} x & = & 0.888... \\ 10x & = & 8.888... \\ 10x - x & & 9x = 8 \\ & & x = \frac{8}{9} \end{array}$$

$$\boxed{x = \frac{8}{9}}$$

2 Amelia says that  $0.\dot{2}9 = \frac{29}{100}$

Explain why Amelia is wrong.

(1 mark)

$0.\dot{2}9$  means  $0.292929...$  where the digits 29 repeat forever

$\frac{29}{100} = 0.29$  exactly, which stops after two decimal places, so it cannot equal the recurring decimal

The correct fraction is  $\frac{29}{99}$

3

Express 0.7 as a fraction.

(2 marks)

$$\text{Let } x = 0.777\ldots$$

$$\text{Then } 10x = 7.777\ldots$$

Multiplying by 10 shifts the decimal one place, so the repeating tails now match.

Subtracting removes that entire matching infinite tail, leaving a whole number

Subtracting

$$10x - x = 7.777\ldots - 0.777\ldots, \text{ so } 9x = 7$$

$$\text{Therefore } x = \frac{7}{9}$$

$$\boxed{\frac{7}{9}}$$

4

Express 0.54 as a fraction in its simplest form.

(3 marks)

$$\text{Let } x = 0.545454\ldots$$

$$\text{Then } 100x = 54.545454\ldots$$

Subtracting

$$100x - x = 54, \text{ so } 99x = 54$$

$$x = \frac{54}{99} = \frac{6}{11} \text{ after dividing top and bottom by 9}$$

$$\boxed{\frac{6}{11}}$$

5

Prove algebraically that the recurring decimal 0.45 can be written as  $\frac{5}{11}$ 

(3 marks)

$$\text{Let } x = 0.454545\ldots$$

Multiply by 100

$$100x = 45.454545\ldots$$

Subtract

$$100x - x = 45.454545\ldots - 0.454545\ldots, \text{ so } 99x = 45$$

$$x = \frac{45}{99}. \text{ Dividing numerator and denominator by 9 gives } x = \frac{5}{11},$$

as required

- 6 Express the recurring decimal  $0.213\overline{}$  as a fraction in its simplest form.  
(The digits 2, 1 and 3 all recur.)

(3 marks)

$$\text{Let } x = 0.213213213\ldots$$

Multiply by 1000

$$1000x = 213.213213\ldots$$

Subtract

$$999x = 213, \text{ so } x = \frac{213}{999}$$

Divide top and bottom by 3

$$x = \frac{71}{333}$$

$$\boxed{\frac{71}{333}}$$

- 7 Write  $1.23\overline{}$  as a mixed number in its simplest form.  
(Only the digit 3 recurs.)

(3 marks)

$$\begin{array}{rcl} x & = & 1.2333\ldots \\ 10x & = & 12.333\ldots \\ 100x & = & 123.333\ldots \\ 100x - 10x & 90x & = 111 \qquad 100x - 10x \\ \div 90 & & \div 90 \\ x & = & \frac{111}{90} \\ & & \div 3 \\ & = & \frac{37}{30} \end{array}$$

$$\boxed{1 \frac{7}{30}}$$

8

Prove algebraically that the recurring decimal  $0.2\overline{8}$  can be written as  $\frac{13}{45}$

(Only the digit 8 recurs.)

(3 marks)

$$\text{Let } x = 0.28888\ldots$$

$$10x = 2.8888\ldots \text{ and } 100x = 28.8888\ldots$$

Subtract

$$100x - 10x = 28.8888\ldots - 2.8888\ldots, \text{ so } 90x = 26$$

$$x = \frac{26}{90} = \frac{13}{45} \text{ after dividing top and bottom by 2,}$$

as required

9

Express the recurring decimal  $0.35\overline{1}$  as a fraction in its simplest form.

(Only the digits 5 and 1 recur.)

(3 marks)

$$\text{Let } x = 0.35151\ldots$$

$$10x = 3.5151\ldots \text{ and } 1000x = 351.5151\ldots$$

Subtract

$$1000x - 10x = 348, \text{ so } 990x = 348$$

$$x = \frac{348}{990} = \frac{58}{165} \text{ after dividing top and bottom by 6}$$

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| $\frac{58}{165}$ |
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10

Work out  $0.1\overline{3} + 0.5$

Give your answer as a fraction in its simplest form.

(3 marks)

$$0.1\overline{3} = \frac{13}{99} \text{ (let } x = 0.1313\ldots, 100x - x = 13, \text{ so } x = \frac{13}{99})$$

$$0.5 = \frac{5}{9} \text{ (let } y = 0.555\ldots, 10y - y = 5, \text{ so } y = \frac{5}{9})$$

$$\frac{5}{9} = \frac{55}{99}, \text{ so the sum is } \frac{13}{99} + \frac{55}{99} = \frac{68}{99}$$

$68 = 4 \times 17$  and  $99 = 9 \times 11$  share no common factor, so  $\frac{68}{99}$  is in

simplest form

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| $\frac{68}{99}$ |
|-----------------|

11

Show algebraically that  $0.7\overline{2} + 0.2\overline{7} = 1$ 

(3 marks)

Let  $x = 0.727272\dots$ , then  $100x = 72.7272\dots$ , so  $99x = 72$  and  $x = \frac{72}{99}$

Let  $y = 0.272727\dots$ , then  $100y = 27.2727\dots$ , so  $99y = 27$  and  $y = \frac{27}{99}$

$$x + y = \frac{72}{99} + \frac{27}{99} = \frac{99}{99} = 1, \text{ as required}$$

12

Work out  $0.1\overline{8} \times 0.4\overline{5}$ 

Give your answer as a fraction in its simplest form.

You must show all your working.

(5 marks)

Let  $x = 0.181818\dots$ , then  $100x = 18.1818\dots$ , so  $99x = 18$  and  $x = \frac{18}{99} = \frac{2}{11}$

Let  $y = 0.454545\dots$ , then  $100y = 45.4545\dots$ , so  $99y = 45$  and  $y = \frac{45}{99} = \frac{5}{11}$

Multiply

$$\frac{2}{11} \times \frac{5}{11} = \frac{10}{121}$$

$10 = 2 \times 5$  and  $121 = 11^2$  share no common factor, so  $\frac{10}{121}$  is in

simplest form

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| $\frac{10}{121}$ |
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