



- 1 Work out the exact value of $\sin 60^\circ \times \cos 30^\circ$.

Give your answer as a fraction.

(2 marks)

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \text{ and } \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{3}{4}$$

$$\boxed{\frac{3}{4}}$$

- 2 In triangle ABC, AB = 8 cm, AC = 6 cm and angle BAC = 60° .

Work out the exact area of triangle ABC. Give your answer in the form $a\sqrt{3} \text{ cm}^2$.

(3 marks)

$$\text{Area} = \frac{1}{2}ab \sin C$$

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 8 \times 6 \times \sin 60^\circ \\ &= 24 \times \frac{\sqrt{3}}{2} \end{aligned}$$

$$\boxed{12\sqrt{3} \text{ cm}^2}$$

- 3 Work out the exact value of $(\sin 30^\circ + \cos 60^\circ) \times \tan 60^\circ$.

(2 marks)

$$\sin 30^\circ = \frac{1}{2} \text{ and } \cos 60^\circ = \frac{1}{2}, \text{ so } \sin 30^\circ + \cos 60^\circ = 1$$

$$\tan 60^\circ = \sqrt{3}$$

$$1 \times \sqrt{3} = \sqrt{3}$$

$$\boxed{\sqrt{3}}$$

- 4 In triangle ABC, AB = 3 cm, AC = 5 cm and BC = $\sqrt{19}$ cm.
Find the size of angle BAC.

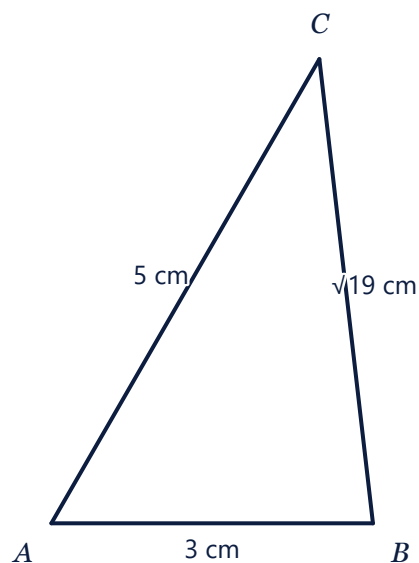


Diagram NOT accurately drawn

(4 marks)

Angle BAC is opposite BC, so use the cosine rule for the angle

$$\cos A = \frac{AB^2 + AC^2 - BC^2}{2 \times AB \times AC}$$

$$\cos A = \frac{9 + 25 - 19}{2 \times 3 \times 5} = \frac{15}{30} = \frac{1}{2}$$

$$\cos A = \frac{1}{2}, \text{ so angle BAC} = 60^\circ$$

$$\boxed{60^\circ}$$

- 5 In triangle ABC, angle ABC = 90° and angle BAC = 30°.

AC = 14 cm.

Without using a calculator, work out the length of BC.

You must show your working.

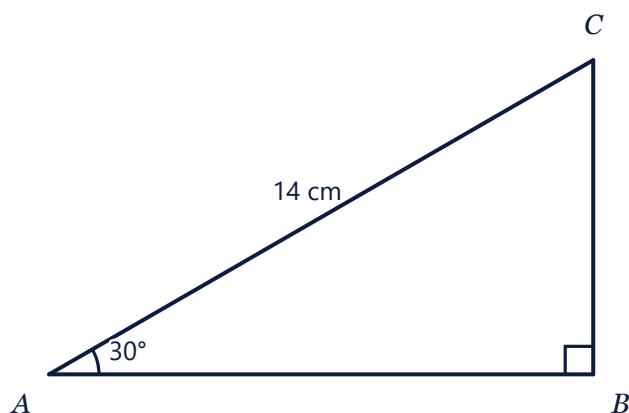


Diagram NOT accurately drawn

(3 marks)

BC is opposite the 30° angle and AC = 14 cm is the hypotenuse,
so $\sin 30^\circ = BC \div 14$

$$\sin 30^\circ = \frac{1}{2}$$

$$BC = 14 \times \frac{1}{2} = 7 \text{ cm}$$

7 cm

- 6 Work out the exact value of $(\sin 45^\circ + \cos 30^\circ)^2$

Give your answer in the form $\frac{a + b\sqrt{6}}{4}$ where a and b are integers.

(3 marks)

$$\begin{aligned}\sin 45^\circ &= \frac{\sqrt{2}}{2} \quad \cos 30^\circ = \frac{\sqrt{3}}{2} \\ &= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \right)^2\end{aligned}$$

Expand

$$\begin{aligned}&= \frac{2}{4} + 2 \times \frac{\sqrt{2} \times \sqrt{3}}{4} + \frac{3}{4} \\ &= \frac{2}{4} + \frac{2\sqrt{6}}{4} + \frac{3}{4}\end{aligned}$$

$$\frac{5 + 2\sqrt{6}}{4}$$

7

In triangle PQR, angle PQR = 90° and angle QPR = 45° .

PQ = 8 cm.

Without using a calculator, work out the exact length of PR.

Give your answer in the form $k\sqrt{2}$.

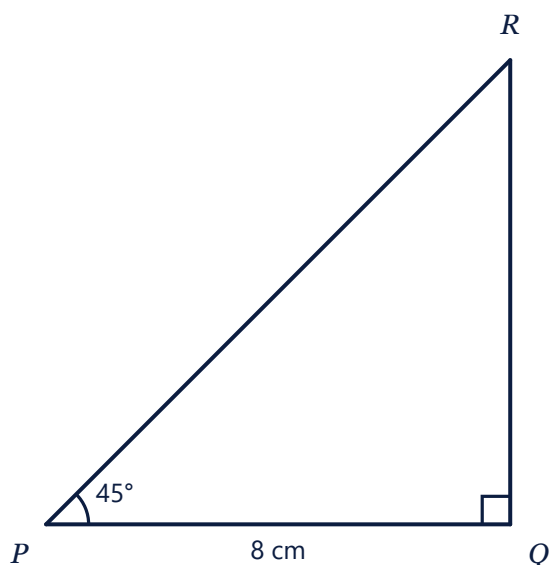


Diagram NOT accurately drawn

(3 marks)

$PQ = 8$ cm is adjacent to the 45° angle and PR is the hypotenuse, so $\cos 45^\circ = 8 \div PR$

$$\cos 45^\circ = \frac{\sqrt{2}}{2}, \text{ so } PR = 8 \div \left(\frac{\sqrt{2}}{2}\right) = 16 \div \sqrt{2}$$

Multiply numerator and denominator by $\sqrt{2}$. This multiplies the fraction by 1, so its value is unchanged, and $\sqrt{2} \times \sqrt{2} = 2$

Rationalise

$$\frac{16}{\sqrt{2}} = \frac{16\sqrt{2}}{2} = 8\sqrt{2} \text{ cm}$$

$$\boxed{8\sqrt{2} \text{ cm}}$$

8

In triangle ABC, angle BAC = 30° , BC = 5 cm and AB = 8 cm.

Work out the exact value of $\sin C$.

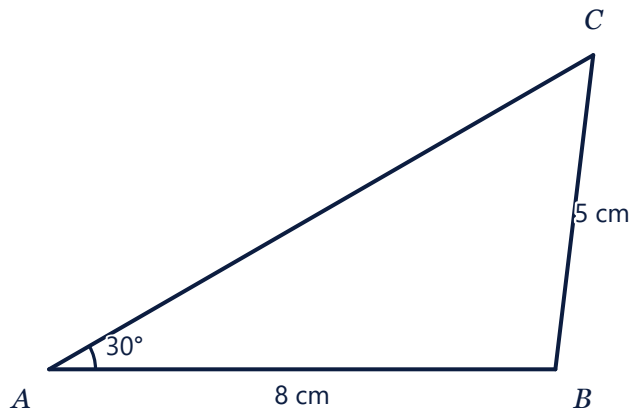


Diagram NOT accurately drawn

(4 marks)

BC is opposite angle A and AB is opposite angle C

Sine rule

$$\frac{\sin C}{AB} = \frac{\sin A}{BC}$$

$$\sin C = 8 \times \sin 30^\circ \div 5$$

$$\sin 30^\circ = \frac{1}{2}, \text{ so } \sin C = 8 \times \frac{1}{2} \div 5 = \frac{4}{5}$$

$$\boxed{\frac{4}{5}}$$

9

In triangle DEF, angle DEF = 90° and angle EDF = 60° .

DE = 5 cm.

Without using a calculator, work out the exact length of EF.

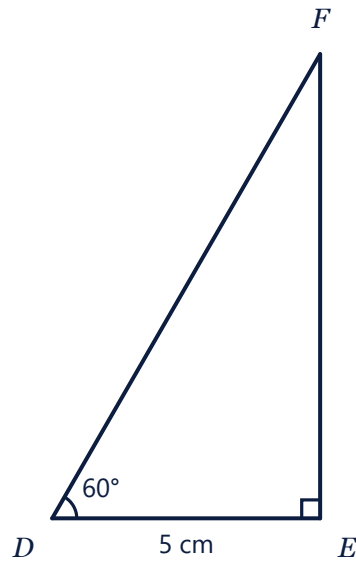


Diagram NOT accurately drawn

(3 marks)

EF is opposite the 60° angle and DE = 5 cm is adjacent to it, so

$$\tan 60^\circ = EF \div 5$$

$$\tan 60^\circ = \sqrt{3}$$

$$EF = 5 \times \sqrt{3} = 5\sqrt{3} \text{ cm}$$

$5\sqrt{3} \text{ cm}$

10

In triangle JKL, angle JKL = 90° .

JK = 6 cm and JL = 12 cm.

Without using a calculator, find the size of angle JLK.

You must show your working.

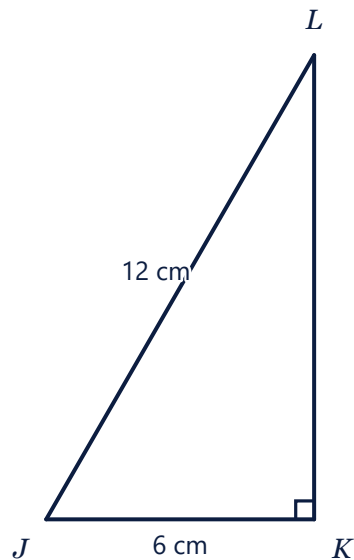


Diagram NOT accurately drawn

(3 marks)

JK = 6 cm is opposite angle JLK and

JL = 12 cm is the hypotenuse

$$\sin \text{JLK} = 6 \div 12 = \frac{1}{2}$$

The exact value $\sin 30^\circ = \frac{1}{2}$, so angle JLK = 30°

30°

11 In triangle ABC, angle ABC = 90°, AB = $\sqrt{5}$ cm and BC = 2 cm. Angle BAC = x.

a. Work out the exact value of sin x and of cos x.

b. Hence find the exact value of tan x, giving your answer with a rational denominator.

(4 marks)

a) Pythagoras for AC

$$AC^2 = (\sqrt{5})^2 + 2^2 = 9$$

$$AC = 3$$

$$\sin x = \frac{\text{opp}}{\text{hyp}} = \frac{2}{3}$$

$$\cos x = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{5}}{3}$$

b)

$$\begin{aligned}\tan x &= \frac{\sin x}{\cos x} = \frac{2}{3} \div \frac{\sqrt{5}}{3} \\ &= \frac{2}{\sqrt{5}}\end{aligned}$$

Rationalise the denominator

$$\tan x = \frac{2\sqrt{5}}{5}$$

12 Work out the exact value of

$$\tan 30^\circ \times \tan 60^\circ + \cos 45^\circ \times \sin 45^\circ$$

Give your answer as a fraction.

(3 marks)

$$\tan 30^\circ = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} \text{ and } \tan 60^\circ = \sqrt{3}, \text{ so } \tan 30^\circ \times \tan 60^\circ = 1$$

$$\cos 45^\circ = \frac{\sqrt{2}}{2} \text{ and } \sin 45^\circ = \frac{\sqrt{2}}{2}, \text{ so } \cos 45^\circ \times \sin 45^\circ = \frac{2}{4} = \frac{1}{2}$$

$$1 + \frac{1}{2} = \frac{3}{2}$$

$$\frac{3}{2}$$

13

In triangle ABC, AB = 6 cm, BC = $4\sqrt{2}$ cm and angle ABC = 45° .

Without using a calculator, work out the exact perimeter of triangle ABC. Give your answer in the form $a + b\sqrt{2} + c\sqrt{5}$.

(5 marks)

Cosine rule to find AC

$$\begin{aligned} AC^2 &= 6^2 + (4\sqrt{2})^2 - 2 \times 6 \times 4\sqrt{2} \times \cos 45^\circ \\ &= 36 + 32 - 48\sqrt{2} \times \frac{\sqrt{2}}{2} \\ &= 68 - 48 \\ &= 20 \end{aligned}$$

$$AC = \sqrt{20} = 2\sqrt{5}$$

Add the three sides

$$6 + 4\sqrt{2} + 2\sqrt{5} \text{ cm}$$

14

An equilateral triangle has side length $2\sqrt{3}$ cm.

Without using a calculator, work out the exact height and the exact area of the triangle.

(4 marks)

Height

$$\begin{aligned} h &= 2\sqrt{3} \times \sin 60^\circ \\ &= 2\sqrt{3} \times \frac{\sqrt{3}}{2} \end{aligned}$$

$$h = 3 \text{ cm}$$

Area = $\frac{1}{2} \times \text{base} \times \text{height}$

$$\text{Area} = \frac{1}{2} \times 2\sqrt{3} \times 3$$

$$3\sqrt{3} \text{ cm}^2$$

15

ABCD is a trapezium in which AB is parallel to DC.

Angle ADC = 90° , angle BCD = 60° , AB = 6 cm and DC = 10 cm.

Without using a calculator, work out the exact area of the trapezium.

Give your answer in the form $k\sqrt{3} \text{ cm}^2$.

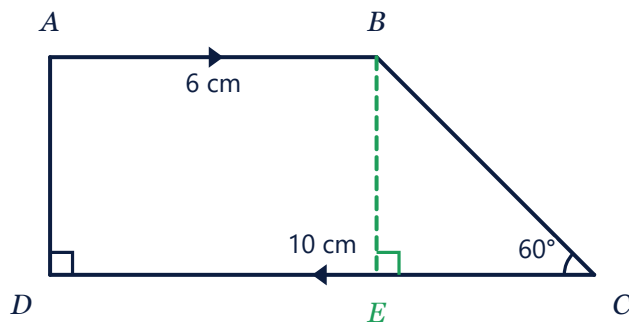


Diagram NOT accurately drawn

(5 marks)

Let h be the height of the trapezium (the length AD)

Drop a perpendicular from B to DC, meeting it at E. Then $DE = AB = 6 \text{ cm}$, so $EC = 10 - 6 = 4 \text{ cm}$

In right-angled triangle BEC, $\tan 60^\circ = BE \div EC$, so $h = BE = 4 \times \tan 60^\circ = 4\sqrt{3} \text{ cm}$

Area of the trapezium = $\frac{1}{2}(AB + DC) \times h = \frac{1}{2}(6 + 10) \times 4\sqrt{3}$

Area = $8 \times 4\sqrt{3} = 32\sqrt{3} \text{ cm}^2$

$32\sqrt{3} \text{ cm}^2$

16

A trapezium has parallel sides of length $(3 + \sqrt{2})$ cm and $(5 + 3\sqrt{2})$ cm.

The perpendicular distance between the parallel sides is $(4 - \sqrt{2})$ cm.

Work out the exact area of the trapezium.

Give your answer in the form $(a + b\sqrt{2})$ cm², where a and b are integers.

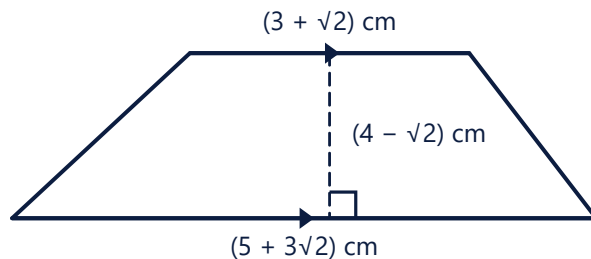


Diagram NOT accurately drawn

(6 marks)

$$\text{Area of a trapezium} = \frac{1}{2} (\text{sum of parallel sides}) \times \text{height}$$

Sum of the parallel sides

$$(3 + \sqrt{2}) + (5 + 3\sqrt{2}) = 8 + 4\sqrt{2}$$

Half of this is $4 + 2\sqrt{2}$

$$\begin{aligned} \text{Area} &= (4 + 2\sqrt{2})(4 - \sqrt{2}) = 16 - 4\sqrt{2} + 8\sqrt{2} - 2 \times 2 \\ &= 16 + 4\sqrt{2} - 4 = 12 + 4\sqrt{2}, \text{ so the area is } (12 + 4\sqrt{2}) \\ &\quad \text{cm}^2 \end{aligned}$$

$$\boxed{12 + 4\sqrt{2}}$$