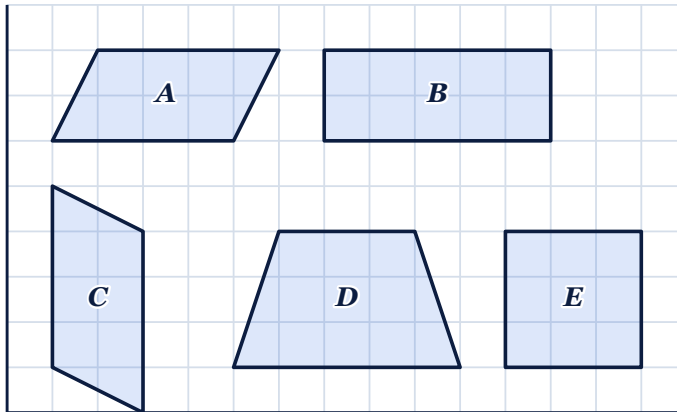




1 The diagram shows five quadrilaterals, labelled A, B, C, D and E, drawn on a centimetre grid.

- Write down the letters of the pair of congruent quadrilaterals.
- Write down the mathematical name of quadrilateral A.



(2 marks)

Congruent shapes are exactly the same shape and size. A and C are identical parallelograms, one rotated, so A and C are the congruent pair
 Quadrilateral A has two pairs of parallel sides with no right angles, so it is a parallelogram

2

Here is some information about four pairs of triangles.

For each pair, write down the condition (SSS, SAS, ASA or RHS) that shows the triangles are congruent, or write "not necessarily congruent".

- Both triangles have two sides of 5 cm and 7 cm, with an angle of 50° between them.
- Both triangles have angles of 40° , 60° and 80° .
- Both triangles have a right angle, a hypotenuse of 13 cm and another side of 5 cm.
- Both triangles have a side of 6 cm with angles of 35° and 70° at each end of it.

(4 marks)

a)

Side, angle between, side

SAS

b)

Same angles, but could be different sizes

Not necessarily congruent

c)

Right angle, hypotenuse, side

RHS

d)

Angle, side between, angle

ASA

3

$PQRS$ is a parallelogram.

Angle $P = (3x + 15)^\circ$ and angle $Q = (2x - 10)^\circ$. Angles P and Q are next to each other on the same side of the parallelogram.

Work out the value of x . Give a reason for each stage of your working.

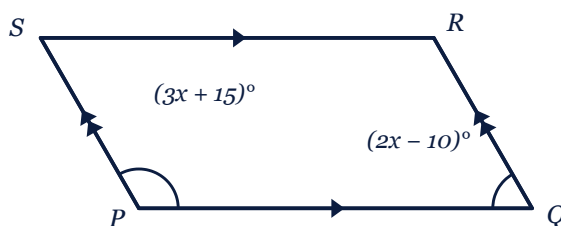


Diagram NOT accurately drawn

(3 marks)

Adjacent angles in a parallelogram add up to 180° (co-interior angles between parallel sides)

$$3x + 15 + 2x - 10 = 180$$

$$5x + 5 = 180, \text{ so } 5x = 175 \text{ and } x = 35$$

Check

$$\text{angle } P = 120^\circ \text{ and angle } Q = 60^\circ, \text{ and } 120 + 60 = 180$$

$$x = 35$$

- 4 AB and CD are parallel lines. The line PQ crosses them at P and Q. One of the angles at P is 62° , as shown.
- Write down the value of x . Give a reason for your answer.
 - Write down the value of z . Give a reason for your answer.
 - Work out the value of y . Give a reason for your answer.

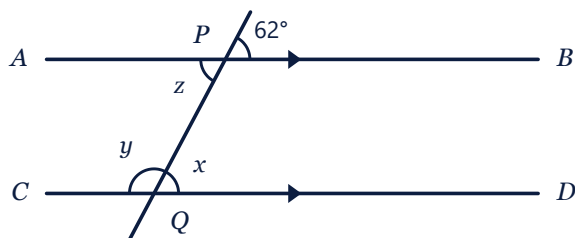


Diagram NOT accurately drawn

(3 marks)

a)

$$x = 62$$

Corresponding angles are equal

b)

$$z = x = 62$$

Alternate angles are equal

c)

$$y = 180 - 62$$

$$y = 118$$

Angles on a straight line add up to 180°

5

AB and CD are parallel straight lines.

A straight line crosses AB at E and crosses CD at F. On line AB, A is to the left of E and B is to the right of E. On line CD, C is to the left of F and D is to the right of F.

Angle AEF = 62° .

Work out the size of angle CFE.

Give a reason for each stage of your working.

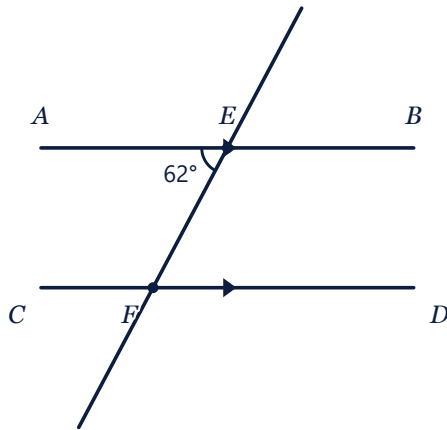


Diagram NOT accurately drawn

(4 marks)

Alternate angles lie between parallel lines on opposite sides of the crossing line. Co-interior angles lie between the parallel lines on the same side of the crossing line

Angle $EFD = 62^\circ$ because angle AEF and angle EFD are alternate angles, and alternate angles are equal

Angle $CFE = 180^\circ - 62^\circ = 118^\circ$ because angles on a straight line add up to 180°

(Alternatively

angle AEF and angle CFE are co-interior angles, so angle $CFE = 180^\circ$

$$\begin{array}{r} - \\ 62^\circ \\ = \\ 118^\circ \end{array}$$

118°

6

PQR is a triangle. The side PQ is extended to the point S .

Angle $RQS = 112^\circ$ and angle $QPR = 44^\circ$.

Show that triangle PQR is isosceles. Give a reason for each stage of your working.

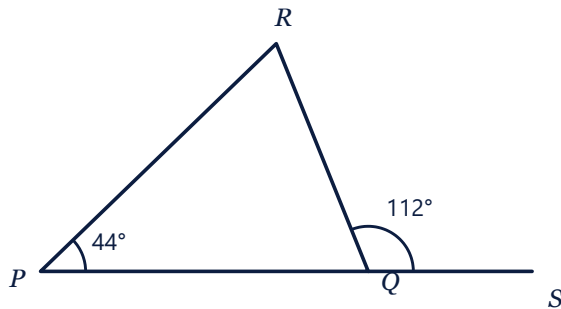


Diagram NOT accurately drawn

(4 marks)

Angle $PQR = 180 - 112 = 68^\circ$ (angles on a straight line add up to 180°)

Angle $PRQ = 180 - 44 - 68 = 68^\circ$ (angles in a triangle add up to 180°)

Angle $PQR = \text{angle } PRQ = 68^\circ$, so triangle PQR has two equal angles

A triangle with two equal angles is isosceles, so triangle PQR is isosceles

- 7 ABC is a triangle. The side BC is extended to the point D.
 Angle BAC = a and angle ABC = b .
 Prove that angle ACD = $a + b$.
 Give a reason for each stage of your working.

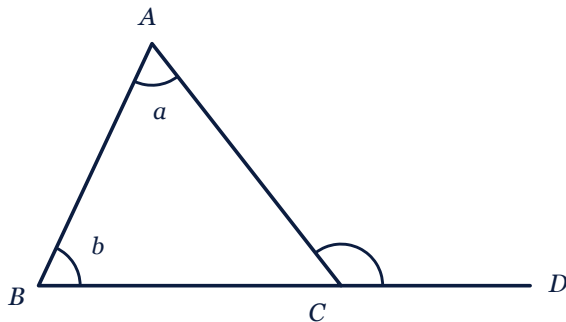


Diagram NOT accurately drawn

(3 marks)

Angles in a triangle add up to 180°

$$ACB = 180 - a - b$$

Angles on a straight line add up to 180°

$$ACD = 180 - ACB$$

$$ACD = 180 - (180 - a - b)$$

$$ACD = a + b$$

$$\boxed{\text{Angle } ACD = a + b}$$

- 8 a. Work out the size of each exterior angle and each interior angle of a regular octagon.
 b. Regular hexagons fit together with no gaps. Regular pentagons do not. Use angles to explain why.

(4 marks)

a)

$$\text{Exterior} = 360 \div 8 = 45^\circ$$

$$\text{Interior} = 180 - 45 = 135^\circ$$

$$\boxed{45^\circ \text{ and } 135^\circ}$$

b)

$$\text{Hexagon: exterior} = 360 \div 6 = 60^\circ$$

$$\text{Interior} = 180 - 60 = 120^\circ$$

$3 \times 120 = 360$, so they fit round a point

$$\text{Pentagon: exterior} = 360 \div 5 = 72^\circ$$

$$\text{Interior} = 180 - 72 = 108^\circ$$

$$360 \div 108 = 3.33...$$

$\boxed{\text{Not a whole number, so pentagons leave a gap}}$

9

ABCDEF is a hexagon.

Angle A = angle D = 90° .

The other four interior angles are all equal.

Work out the size of angle B.

(4 marks)

The interior angles of a hexagon add up to $(6 - 2) \times 180^\circ = 720^\circ$

The four equal angles add up to $720 - 90 - 90 = 540^\circ$

Angle B = $540 \div 4 = 135^\circ$

135°

10

AB and CD are parallel straight lines. A straight line crosses them both.

Two co-interior angles between the parallel lines are $(3x + 20)^\circ$ and $(2x - 15)^\circ$.

Work out the size of the smaller of these two angles.

You must show all your working.

(5 marks)

Co-interior angles between parallel lines add up to 180° , so $(3x + 20) + (2x - 15) = 180$

$5x + 5 = 180$, so $5x = 175$ and $x = 35$

The two angles are $3(35) + 20 = 125^\circ$ and $2(35) - 15 = 55^\circ$

Check

$125^\circ + 55^\circ = 180^\circ$. The smaller angle is 55°

55°

11

In triangle ABC, $AB = AC$.

The point D lies on BC so that AD bisects angle BAC.

Prove that triangle ABD is congruent to triangle ACD.

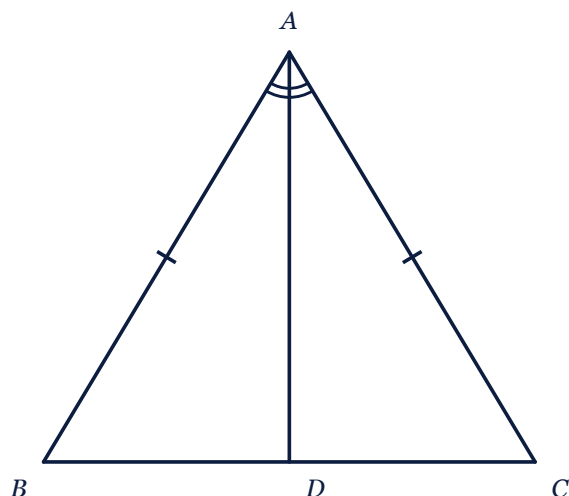


Diagram NOT accurately drawn

(3 marks)

$$AB = AC \text{ (given)}$$

Angle BAD = angle CAD because AD bisects angle BAC (given)

AD is common to both triangles, so $AD = AD$

In each triangle the equal angle is the included angle between the two equal sides (AB and AD in triangle ABD, AC and AD in triangle ACD)

So triangle ABD is congruent to triangle ACD (SAS)

12

AB and CD are parallel straight lines, with AB above CD.

E lies on AB and G lies on CD. F is a point between the two lines, to the right of both E and G, so that EFG forms a zigzag.

Angle BEF = 38° and angle DGF = 44° , where B is to the right of E and D is to the right of G.

Work out the size of angle EFG.

Give a reason for each stage of your working.

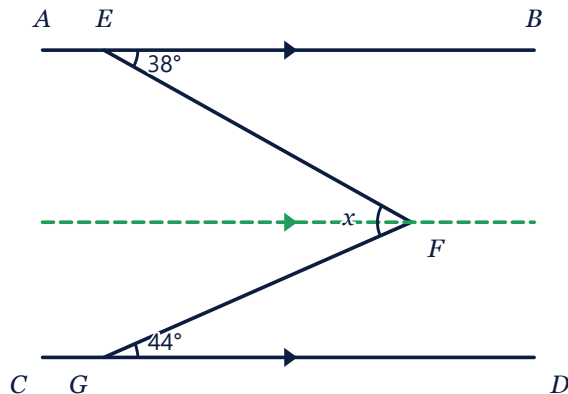


Diagram NOT accurately drawn

(5 marks)

Draw a line through F parallel to AB and CD, splitting angle EFG into an upper part and a lower part

The upper part of angle EFG = 38° because it is alternate to angle BEF, and alternate angles are equal

The lower part of angle EFG = 44° because it is alternate to angle DGF, and alternate angles are equal

$$\text{Angle EFG} = 38^\circ + 44^\circ = 82^\circ$$

82

EXTENSION QUESTIONS

13

ABCDEF is a regular hexagon. ABGH is a square.

The hexagon and the square have the same side length and share the side AB.

F is joined to H.

a. Work out the size of angle FAH.

b. Work out the size of angle AFH.

Give a reason for each stage of your working.

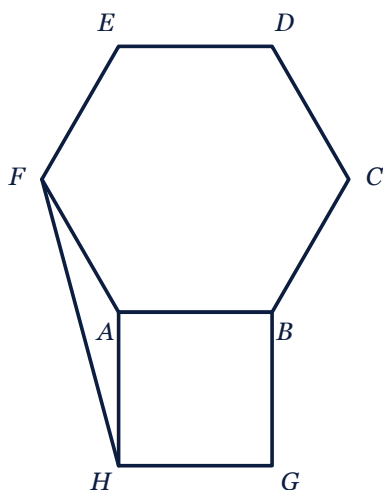


Diagram NOT accurately drawn

(5 marks)

a)

$$\text{Hexagon: } (6 - 2) \times 180 = 720$$

$$720 \div 6 = 120^\circ$$

$$\text{Square: } 90^\circ$$

Angles around a point add up to 360°

$$\text{FAH} = 360 - 120 - 90$$

$$\text{FAH} = 150^\circ$$

$$\boxed{150^\circ}$$

b)

AF = AH, so triangle AFH is isosceles

$$\text{AFH} = \frac{180 - 150}{2}$$

$$\text{AFH} = 15^\circ$$

$$\boxed{15^\circ}$$