



- 1** The diagram shows a right-angled triangle PQR.
- Write down the name of the side that is the hypotenuse.
  - Show that the three lengths fit Pythagoras' theorem.

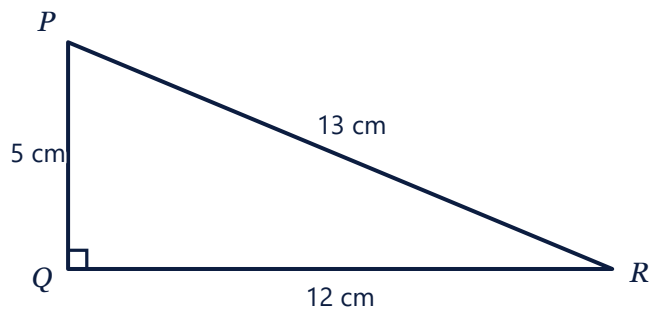


Diagram NOT accurately drawn

(2 marks)

a)

Opposite the right angle

**PR**

b)

$$5^2 + 12^2 = 25 + 144$$

$$= 169$$

$$13^2 = 169$$

$$5^2 + 12^2 = 13^2$$

2

Triangle ABC is a right-angled triangle with the right angle at B.

AB = 6 cm and BC = 8 cm.

Work out the length of AC.

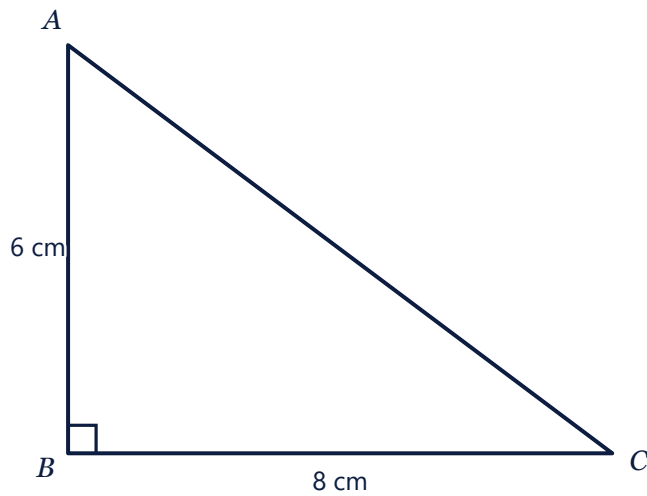


Diagram NOT accurately drawn

(2 marks)

The hypotenuse is the side opposite the right angle and is the longest side.

Pythagoras relates the areas of squares on the three sides: the two smaller areas add to the largest. It applies to right-angled triangles

AC is the hypotenuse, so use Pythagoras' theorem

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 6^2 + 8^2 = 36 + 64 = 100$$

$$AC = \sqrt{100} = 10 \text{ cm}$$

10 cm

- 3 Triangle PQR is a right-angled triangle with the right angle at Q.  
PQ = 7 cm and QR = 9 cm.  
Work out the length of PR.  
Give your answer correct to 1 decimal place.

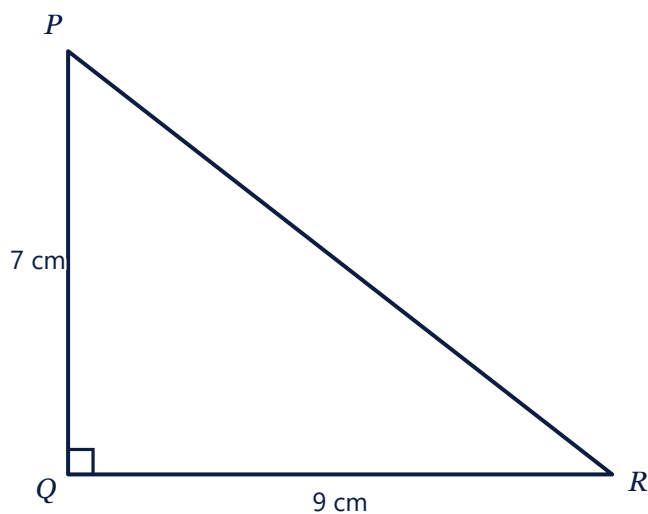


Diagram NOT accurately drawn

(2 marks)

PR is the hypotenuse, so  $PR^2 = PQ^2 + QR^2$

$$PR^2 = 7^2 + 9^2 = 49 + 81 = 130$$

$$PR = \sqrt{130} = 11.401... = 11.4 \text{ cm (1 decimal place)}$$

11.4 cm

4

The diagram shows a right-angled triangle.

Work out the length  $x$ .

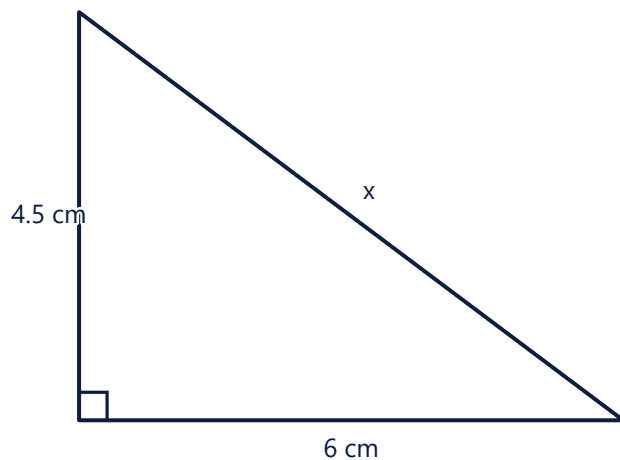


Diagram NOT accurately drawn

(2 marks)

$$x^2 = 4.5^2 + 6^2$$

$$x^2 = 20.25 + 36$$

$$x^2 = 56.25$$

$$\sqrt{\phantom{x}} \quad x = \sqrt{56.25} \quad \sqrt{\phantom{x}}$$

$$x = 7.5$$

7.5 cm

5 The diagrams show two right-angled triangles.

- Work out the length  $x$ .
- Work out the length  $y$ . Give your answer correct to 1 decimal place.

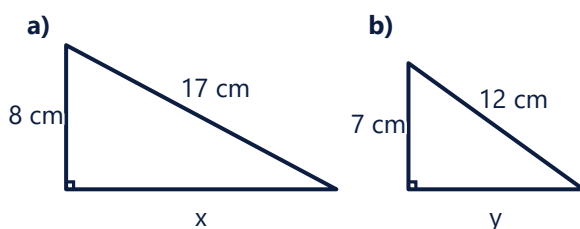


Diagram NOT accurately drawn

(3 marks)

a)

Shorter side, so subtract

$$\begin{aligned}x^2 &= 17^2 - 8^2 \\x^2 &= 289 - 64 \\x^2 &= 225 \\x &= 15\end{aligned}$$

15 cm

b)

$$\begin{aligned}y^2 &= 12^2 - 7^2 \\y^2 &= 144 - 49 \\y^2 &= 95 \\y &= 9.746...\end{aligned}$$

9.7 cm

6 A rectangle is 9 cm long and 6 cm wide.

Work out the length of a diagonal of the rectangle.

Give your answer correct to 1 decimal place.

(2 marks)

The diagonal is the hypotenuse of a right-angled triangle with sides 9 cm and 6 cm

$$\text{diagonal}^2 = 9^2 + 6^2 = 81 + 36 = 117$$

$$\text{diagonal} = \sqrt{117} = 10.816... = 10.8 \text{ cm (1 decimal place)}$$

10.8 cm

7

a. A is the point (1, 2) and B is the point (7, 10). Work out the length of the line segment AB.

b. C is the point (-3, 4) and D is the point (2, -8). Work out the length of the line segment CD.

(3 marks)

a)

$$\text{Across: } 7 - 1 = 6$$

$$\text{Up: } 10 - 2 = 8$$

$$AB^2 = 6^2 + 8^2$$

$$AB^2 = 36 + 64$$

$$AB^2 = 100$$

$$\sqrt{\quad} \quad AB = 10 \quad \sqrt{\quad}$$

10 units

b)

$$\text{Across: } 2 - (-3) = 5$$

$$\text{Down: } 4 - (-8) = 12$$

$$CD^2 = 5^2 + 12^2$$

$$CD^2 = 25 + 144$$

$$CD^2 = 169$$

$$\sqrt{\quad} \quad CD = 13 \quad \sqrt{\quad}$$

13 units

8

A triangle has sides of length 10 cm, 24 cm and 26 cm.

Show that the triangle is a right-angled triangle.

(3 marks)

$$10^2 + 24^2 = 100 + 576 = 676$$

$$26^2 = 676$$

Since  $10^2 + 24^2 = 26^2$ , the sides satisfy Pythagoras' theorem, so the triangle is right-angled (the right angle is opposite the 26 cm side)

9

A rectangular football pitch is 100 m long and 64 m wide.

Sam runs from one corner to the opposite corner along the diagonal.

Tom runs from the same corner to the same opposite corner along two sides of the pitch.

How much further does Tom run than Sam? Give your answer correct to 1 decimal place.

(3 marks)

$$d^2 = 100^2 + 64^2$$

$$d^2 = 10\,000 + 4096$$

$$d^2 = 14\,096$$

$$\sqrt{\phantom{x}} \quad d = 118.727... \quad \sqrt{\phantom{x}}$$

$$\text{Tom: } 100 + 64 = 164 \text{ m}$$

$$164 - 118.727... = 45.272...$$

$$\boxed{45.3 \text{ m}}$$

10

ABCD is a trapezium in which AB is parallel to DC.

Angle ADC = angle DAB =  $90^\circ$ .

AB = 9 cm, AD = 12 cm and DC = 14 cm.

Work out the perimeter of the trapezium.

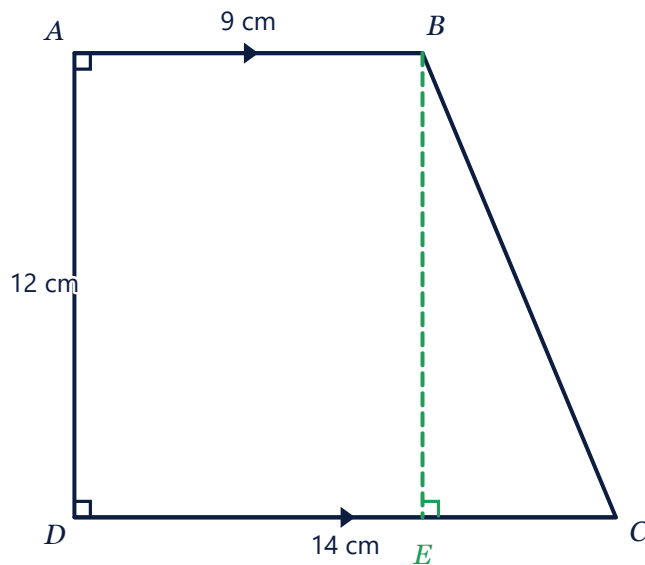


Diagram NOT accurately drawn

(4 marks)

Draw the line from B perpendicular to DC, meeting DC at E. Then  $DE = AB = 9$  cm and  $BE = AD = 12$  cm

$$EC = 14 - 9 = 5 \text{ cm}$$

$$BC^2 = 12^2 + 5^2 = 144 + 25 = 169, \text{ so } BC = 13 \text{ cm}$$

$$\text{Perimeter} = 9 + 13 + 14 + 12 = 48 \text{ cm}$$

48 cm

11

The diagram shows an isosceles triangle with two sides of 13 cm and a base of 10 cm.

- Work out the perpendicular height  $h$  of the triangle.
- Work out the area of the triangle.

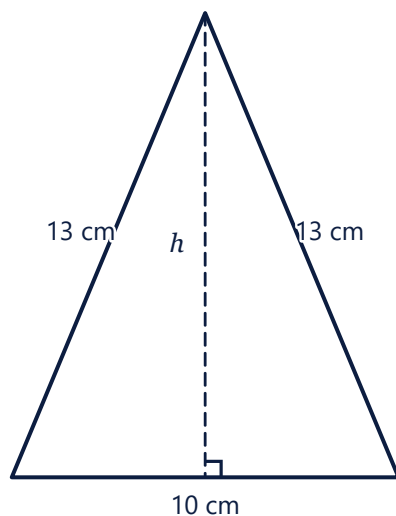


Diagram NOT accurately drawn

(4 marks)

a)

Split into two right-angled triangles

Half the base:  $10 \div 2 = 5$

$$h^2 = 13^2 - 5^2$$

$$h^2 = 169 - 25$$

$$h^2 = 144$$

$$\sqrt{\quad} \quad h = 12 \quad \sqrt{\quad}$$

12 cm

b)

$$\text{Area} = \frac{1}{2} \times 10 \times 12$$

$$\text{Area} = 60$$

60 cm<sup>2</sup>

12 A field ABCD is a trapezium in which AB is parallel to DC.

Angle DAB = angle ADC =  $90^\circ$ .

AB = 25 m, AD = 15 m and DC = 33 m.

Joe wants to put a fence around the whole perimeter of the field.

Fencing is sold in 12 m rolls. Each roll costs £19.50.

Work out the total cost of the rolls of fencing Joe must buy.

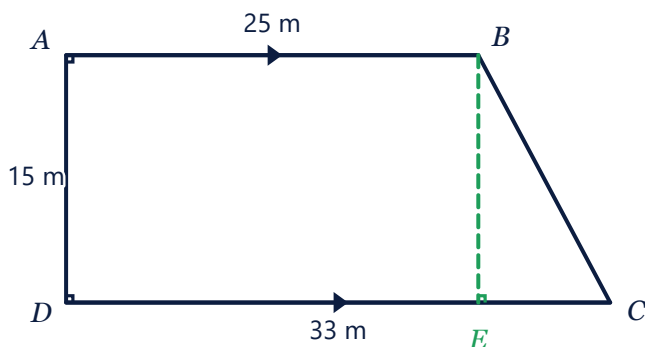


Diagram NOT accurately drawn

(5 marks)

Draw the line from B perpendicular to DC, meeting DC at E. Then  $EC = 33 - 25 = 8$  m and  $BE = 15$  m

$$BC^2 = 8^2 + 15^2 = 64 + 225 = 289, \text{ so } BC = 17 \text{ m}$$

$$\text{Perimeter} = 25 + 17 + 33 + 15 = 90 \text{ m}$$

$$90 \div 12 = 7.5, \text{ so Joe must buy 8 rolls}$$

$$\text{Total cost} = 8 \times £19.50 = £156$$

£156

#### EXTENSION QUESTIONS

13

ABC is a right-angled triangle. AB = 12 cm, BC = 5 cm and AC = 13 cm. Angle BAC =  $\theta$

- Write down which side is the hypotenuse, which side is opposite  $\theta$  and which side is adjacent to  $\theta$
- Write  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$  as fractions.

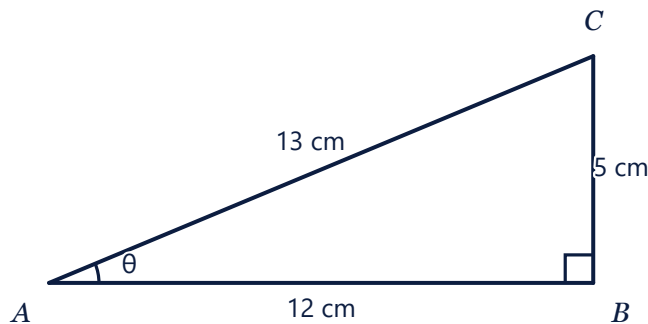


Diagram NOT accurately drawn

(4 marks)

a)

hypotenuse: opposite the right angle

opposite: across from  $\theta$

adjacent: next to  $\theta$

hyp AC, opp BC, adj AB

b) SOH CAH TOA

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{5}{13}$$

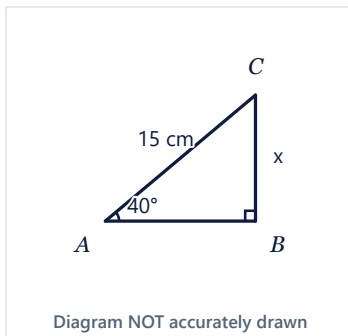
$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{12}{13}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{5}{12}$$

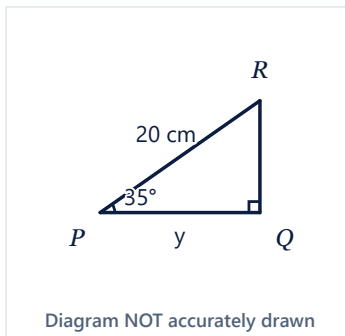
$$\sin \theta = \frac{5}{13}, \cos \theta = \frac{12}{13}, \tan \theta = \frac{5}{12}$$

14

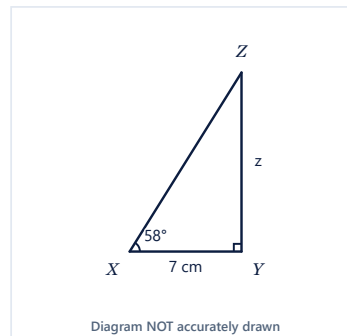
Work out the length of the side marked with a letter in each triangle. Give each answer correct to 3 significant figures.



(a)



(b)



(c)

(4 marks)

a) opp and hyp: SOH

$$\sin 40^\circ = \frac{x}{15}$$

$$\begin{aligned} \times 15 \quad x &= 15 \times \sin 40^\circ \quad \times 15 \\ x &= 9.641... \end{aligned}$$

$$\boxed{x = 9.64 \text{ cm}}$$

b) adj and hyp: CAH

$$\cos 35^\circ = \frac{y}{20}$$

$$\begin{aligned} \times 20 \quad y &= 20 \times \cos 35^\circ \quad \times 20 \\ y &= 16.38... \end{aligned}$$

$$\boxed{y = 16.4 \text{ cm}}$$

c) opp and adj: TOA

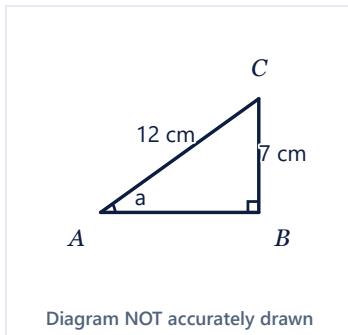
$$\tan 58^\circ = \frac{z}{7}$$

$$\begin{aligned} \times 7 \quad z &= 7 \times \tan 58^\circ \quad \times 7 \\ z &= 11.20... \end{aligned}$$

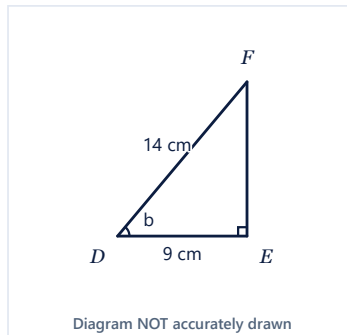
$$\boxed{z = 11.2 \text{ cm}}$$

15

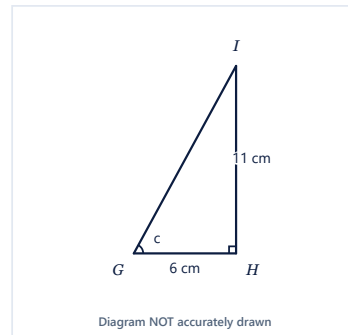
Work out the size of the angle marked  $a$ ,  $b$  and  $c$ . Give each answer correct to 1 decimal place.



(a)



(b)



(c)

(4 marks)

a) opp and hyp: SOH

$$\sin a = \frac{7}{12}$$

$$a = \sin^{-1}\left(\frac{7}{12}\right)$$

$$a = 35.68\dots$$

$$\boxed{35.7^\circ}$$

b) adj and hyp: CAH

$$\cos b = \frac{9}{14}$$

$$b = \cos^{-1}\left(\frac{9}{14}\right)$$

$$b = 49.99\dots$$

$$\boxed{50.0^\circ}$$

c) opp and adj: TOA

$$\tan c = \frac{11}{6}$$

$$c = \tan^{-1}\left(\frac{11}{6}\right)$$

$$c = 61.38\dots$$

$$\boxed{61.4^\circ}$$

16

A wheelchair ramp is 4.5 m long. It makes an angle of  $8^\circ$  with the horizontal ground.

- Work out the height,  $h$ , that the ramp rises. Give your answer correct to 3 significant figures.
- Work out the horizontal distance,  $d$ , that the ramp covers. Give your answer correct to 3 significant figures.
- A safety rule says a ramp should rise no more than 1 m for every 12 m across. Does this ramp follow the rule? Show how you decide.

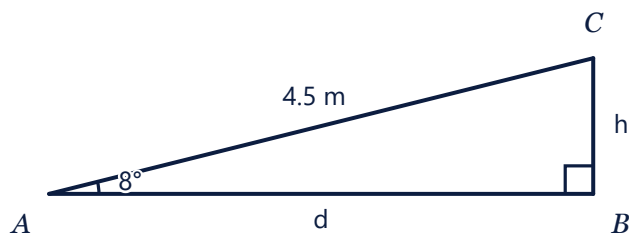


Diagram NOT accurately drawn

(4 marks)

a) opp and hyp: SOH

$$\sin 8^\circ = \frac{h}{4.5}$$

$\times 4.5$

$$h = 4.5 \times \sin 8^\circ$$

$\times 4.5$

$$h = 0.6262\dots$$

0.626 m

b) adj and hyp: CAH

$$\cos 8^\circ = \frac{d}{4.5}$$

$\times 4.5$

$$d = 4.5 \times \cos 8^\circ$$

$\times 4.5$

$$d = 4.456\dots$$

4.46 m

c) rise for 12 m across

$$\text{rise} = 0.6262\dots \div 4.456\dots \times 12$$

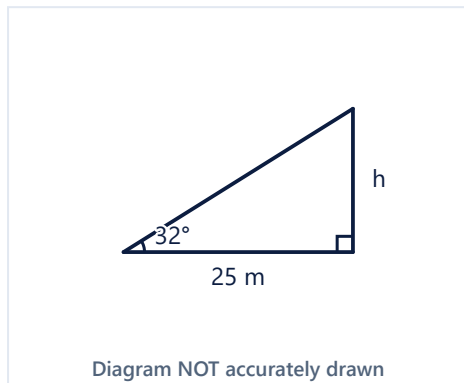
$$\text{rise} = 1.69 \text{ m}$$

$$1.69 \text{ m} > 1 \text{ m}$$

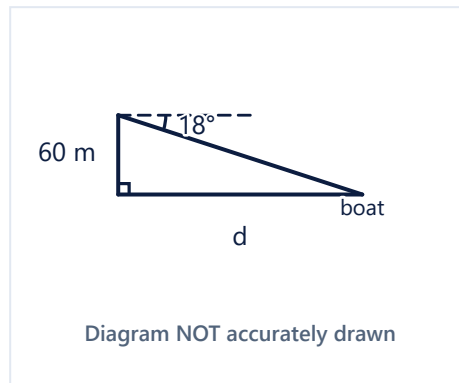
No, it is too steep

17

- a. Sofia stands 25 m from the foot of a tree, on level ground. The angle of elevation of the top of the tree from where she stands is  $32^\circ$ . Work out the height of the tree,  $h$ . Give your answer correct to 3 significant figures.
- b. The top of a cliff is 60 m above the sea. From the top of the cliff, the angle of depression of a boat is  $18^\circ$ . Work out the distance,  $d$ , of the boat from the foot of the cliff. Give your answer correct to 3 significant figures.



(a)



(b)

(4 marks)

a) opp and adj: TOA

$$\tan 32^\circ = \frac{h}{25}$$

$$\begin{aligned} \times 25 \quad h &= 25 \times \tan 32^\circ \quad \times 25 \\ h &= 15.62... \end{aligned}$$

15.6 m

b) angle at the boat is also  $18^\circ$  (alternate)

opp 60, adj d: TOA

$$\tan 18^\circ = \frac{60}{d}$$

$$\begin{aligned} \times d \quad d \times \tan 18^\circ &= 60 \quad \times d \\ \div \tan 18^\circ \quad d &= \frac{60}{\tan 18^\circ} \quad \div \tan 18^\circ \\ d &= 184.6... \end{aligned}$$

185 m

18 In triangle ABC, angle ABC =  $90^\circ$  and angle BAC =  $30^\circ$ .

AC = 14 cm.

Without using a calculator, work out the length of BC.

You must show your working.

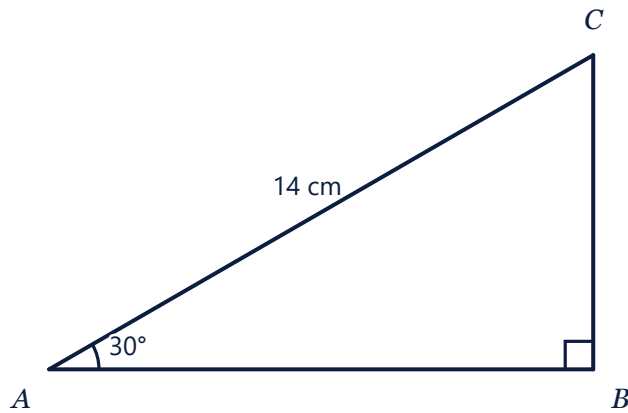


Diagram NOT accurately drawn

(3 marks)

BC is opposite the  $30^\circ$  angle and AC = 14 cm is the hypotenuse,  
so  $\sin 30^\circ = BC \div 14$

$$\sin 30^\circ = \frac{1}{2}$$

$$BC = 14 \times \frac{1}{2} = 7 \text{ cm}$$

7 cm

19

In triangle ABC, angle BAC =  $35^\circ$ , angle ABC =  $78^\circ$  and BC = 9.4 cm.

Work out the length of AC. Give your answer correct to 3 significant figures.

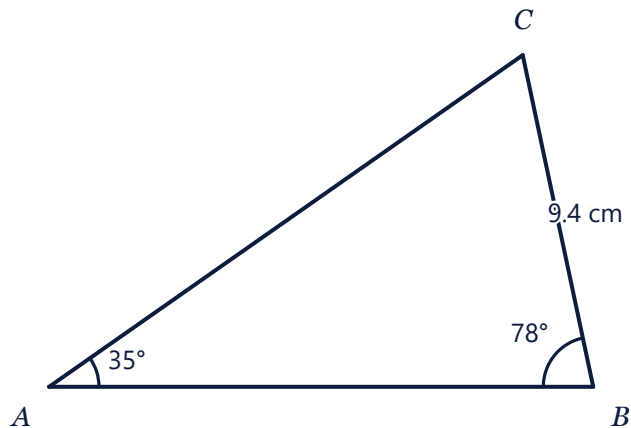


Diagram NOT accurately drawn

(3 marks)

AC is opposite angle B and BC is opposite angle A

Sine rule

$$\frac{AC}{\sin B} = \frac{BC}{\sin A}$$

$$AC = 9.4 \times \sin 78^\circ \div \sin 35^\circ$$

$$AC = 9.4 \times 0.97814... \div 0.57357... = 16.030..$$

$$AC = 16.0 \text{ cm (3 sf)}$$

$$\boxed{16.0 \text{ cm}}$$

20

In triangle ABC,  $AB = 8$  cm,  $AC = 11$  cm and angle  $BAC = 62^\circ$ .

Work out the length of BC. Give your answer correct to 3 significant figures.

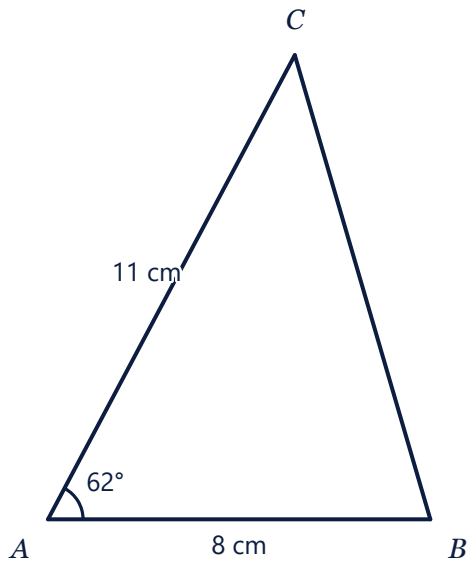


Diagram NOT accurately drawn

(4 marks)

We know two sides and the angle between them. The cosine rule finds the opposite side; unlike Pythagoras, it works even when the angle is not  $90^\circ$ . Use degree mode and keep the full value until the final rounding

Cosine rule

$$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos A$$

$$BC^2 = 8^2 + 11^2 - 2 \times 8 \times 11 \times \cos 62^\circ$$

$$BC^2 = 64 + 121 - 176 \times 0.46947... = 102.37..$$

$$BC = \sqrt{102.37...} = 10.117..$$

$$BC = 10.1 \text{ cm (3 s.f.)}$$

$$\boxed{10.1 \text{ cm}}$$