



1 Here are the first four terms of a sequence.

3 6 12 24

- Write down the next term of the sequence.
- Write down the term-to-term rule of the sequence.

(2 marks)

Each term is double the term before

$$3 \times 2 = 6, 6 \times 2 = 12, 12 \times 2 = 24$$

The next term is $24 \times 2 = 48$

The term-to-term rule is multiply by 2

2 Here are the first four terms of a sequence.

14 11 8 5

- Work out the next two terms of the sequence.
- Write down the term-to-term rule of the sequence.

(3 marks)

The terms go down by 3 each time: 14, 11, 8, 5

Next term

$$5 - 3 = 2$$

The term after that

$$2 - 3 = -1$$

The term-to-term rule is subtract 3

3 Here are four sequences.

A 4 9 14 19

B 1 4 9 16

C 2 6 18 54

D 1 1 2 3 5

For each sequence, write down

- the type of sequence: arithmetic, geometric, square numbers or Fibonacci-type
- the next term.

(4 marks)

A: +5 each time

Arithmetic, 24

B: $1^2, 2^2, 3^2, 4^2$

Square numbers, 25

C: $\times 3$ each time

Geometric, 162

D: add the two before

Fibonacci-type, 8

- 4 The n th term of a sequence is $6n - 1$

Write down the first three terms of the sequence.

(2 marks)

$$\text{When } n = 1: 6(1) - 1 = 5$$

$$\text{When } n = 2: 6(2) - 1 = 11$$

$$\text{When } n = 3: 6(3) - 1 = 17$$

5, 11, 17

- 5 a. The n th term of a sequence is $20 - 3n$

Write down the first three terms and work out the 10th term.

- b. The n th term of another sequence is $n^2 + 2$

Write down the first three terms.

(3 marks)

a)

$$n = 1: 20 - 3 \times 1 = 17$$

$$n = 2: 20 - 3 \times 2 = 14$$

$$n = 3: 20 - 3 \times 3 = 11$$

$$n = 10: 20 - 3 \times 10 = -10$$

17, 14, 11 and -10

b)

$$n = 1: 1^2 + 2 = 3$$

$$n = 2: 2^2 + 2 = 6$$

$$n = 3: 3^2 + 2 = 11$$

3, 6, 11

- 6 Here are the first four terms of an arithmetic sequence.

5 9 13 17

Find an expression, in terms of n , for the n th term of the sequence.

(2 marks)

n is the position in the sequence, starting with $n = 1$. The n th-term rule lets you calculate a term directly from its position

The common difference is 4, so the n th term contains $4n$

The sequence $4n$ gives 4, 8, 12, 16, which is 1 less than each term

The n th term is $4n + 1$

Check

$$\text{when } n = 3, 4(3) + 1 = 13$$

$4n + 1$

7 Here are the first four terms of an arithmetic sequence.

20 17 14 11

Find an expression, in terms of n , for the n th term of the sequence.

(2 marks)

The common difference is -3 , so the n th term contains $-3n$

The sequence $-3n$ gives $-3, -6, -9, -12$, which is 23 less than each term

The n th term is $23 - 3n$

Check

when $n = 2$, $23 - 6 = 17$

$$23 - 3n$$

8 Kwame makes patterns from square tiles.

Pattern 1 uses 6 tiles. Pattern 2 uses 10 tiles. Pattern 3 uses 14 tiles.

The number of tiles goes up by the same amount each time.

- Find an expression, in terms of n , for the number of tiles in Pattern n .
- Work out the number of tiles in Pattern 20
- Kwame says, "One of my patterns uses exactly 100 tiles." Is Kwame right? Show how you decide.

(4 marks)

a)

6, 10, 14

Difference = $+4$, so $4n$

$4n$: 4, 8, 12

Add 2 to get 6, 10, 14

$$4n + 2$$

b)

$$4 \times 20 + 2 = 82$$

82 tiles

c)

$$\begin{array}{rcl} 4n + 2 & = & 100 \\ - & & \\ 4n & = & 98 \\ \div 4 & & \div 4 \\ n & = & 24.5 \end{array}$$

Not a whole number

No, Kwame is wrong

9 The n th term of a sequence is $5n - 3$

- Work out the 12th term of the sequence.
- Is 122 a term of the sequence? You must show how you get your answer.

(3 marks)

$$a) 5(12) - 3 = 60 - 3 = 57$$

$$b) \text{ Solve } 5n - 3 = 122: 5n = 125, \text{ so } n = 25$$

$n = 25$ is a whole number, so 122 is the 25th term of the sequence

10 Here are the first four terms of an arithmetic sequence.

8 15 22 29

- Find an expression, in terms of n , for the n th term of the sequence.
- Explain why 150 is not a term of the sequence.

(4 marks)

a) The common difference is 7, and $7n$ gives 7, 14, 21, 28, which is 1 less than each term

The n th term is $7n + 1$

b) If $7n + 1 = 150$ then $7n = 149$

149 divided by 7 is not a whole number, so 150 is not a term

11 Here are the first five terms of a quadratic sequence.

2 7 14 23 34

Find an expression, in terms of n , for the n th term of this sequence.

(2 marks)

For a quadratic sequence $an^2 + bn + c$, the second difference is $2a$. The n^2 terms 1, 4, 9, 16, ... have first differences 3, 5, 7, ... and second difference 2; multiplying by a makes it $2a$. The linear part $bn + c$ adds no second difference

First differences

5, 7, 9, 11. Second difference: 2, so the n^2 coefficient is 2 divided by 2 = 1

Subtract n^2 (1, 4, 9, 16, 25) from the sequence: 1, 3, 5, 7, 9, which has n th term $2n - 1$

So the n th term is $n^2 + 2n - 1$

Check

$n = 1$ gives $1 + 2 - 1 = 2$; $n = 2$ gives $4 + 4 - 1 = 7$; $n = 3$ gives $9 + 6 - 1 = 14$; $n = 4$ gives $16 + 8 - 1 = 23$; $n = 5$ gives $25 + 10 - 1 = 34$

$n^2 + 2n - 1$

EXTENSION QUESTIONS

12

Here are the first five terms of a quadratic sequence.

4 11 22 37 56

- Find an expression, in terms of n , for the n th term of the sequence.
- Work out the 10th term of the sequence.

(5 marks)

a)

4, 11, 22, 37, 56

1st differences: 7, 11, 15, 19

2nd differences: 4, 4, 4

 $4 \div 2 = 2$, so $2n^2$ $2n^2$: 2, 8, 18, 32, 50Sequence - $2n^2$: 2, 3, 4, 5, 62, 3, 4, 5, 6 is $n + 1$

$$2n^2 + n + 1$$

b)

$$2 \times 10^2 + 10 + 1$$

$$= 200 + 10 + 1$$

$$211$$